Estimate Shear Velocity Based on Dry P-wave and Shear Modulus Relationship

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Summary

We have studied relationship of bulk and shear modulus for consolidated sandstone and poorly consolidated sands. Applying the Gassmann's equation, we have developed a method (G-method) to estimate shear velocity based on an empirical relationship between P-wave and shear modulus of dry sands and sandstones.

Introduction

One of the challenges in detection of reservoir fluids, such as fluid substitution in AVO analysis, is that there is no shear velocity data, or quality of shear velocity data is questionable. Many efforts have been made to estimate shear velocities. P and S-wave velocity data of sandstone samples show tight correlation in water-saturated sandstones (Han, 1986),

$$V_s = 0.79 * V_p - 0.79 . \qquad (1)$$

Measured velocity data has revealed that although both Vp and Vs of sandstones varies systematically to porosity, clay content and differential pressure, they correlate simply as shown in equation (1). No apparent effects of porosity, clay content and differential pressure on equation (1) suggests that, in first order, there is a simple linear relationship of Vp and Vs to these parameters. The simple Vp-Vs relation seems to tie with texture of sand aggregates.

The equation (1) is similar to the mud-line formula derived from log data by Castagna et al, (1985). Greenburg -Castagna (1992) and Xu -White (1996) have developed different methods to predict shear velocity. In this paper, we review relationship of bulk and shear modulus and apply the Gassmann's equation to predict shear modulus.

Although velocities and modulus correlate to each other, their physical meaning is different.

$$\left(\frac{V_p}{V_s} \right)^2 = \frac{M_s}{\mu} = \frac{M_d + \Delta K_d}{\mu} ,$$

$$M_d = K_d + 4/3 * \mu , \qquad (2)$$

where M_d is dry P-wave modulus, μ is shear modulus and ΔK_d is an increment of bulk modulus due to fluid saturation.. We separate saturated P-wave modulus into dry P-wave modulus and fluid effect ΔK_d. Clearly, both dry bulk and shear modulus correlates intrinsically:

$$\frac{K_d}{\mu_d} = \frac{V_{pd}^2}{V_{sd}^2} - \frac{4}{3} . \qquad (3)$$

Grain contact model

Grain contact model is used to study elastic properties of clastic rocks, especially loose sands. Many models have been proposed with different assumptions (Hertz-Mindlin, 1949; Walton, 1987; Dvorkin and Nur, 1996) and show different effect of pressure, porosity, mineral property, grain contact, cement and so on. These models provide tools to reveal physical properties of sand aggregates. First, we examine dry sand properties.

Figure 1 shows the result for the Hertz-Mindlin model. For a sand aggregate with 33% porosity, the dry bulk (K_d) and shear (μ) modulus increases with pressure (Figure 1a), but the dry K/µ ratio is independent of pressure around 0.7 (Figure 1b). Similarly, both the bulk and shear modulus decreases with increasing porosity from 20 to 40%, but the K/µ ratio remains a constant (Figure 1c.). Figure 1d shows that K/ µ ratio increases slightly with the mineral K_g/μ_g ratio. For pure quartz, K_g/μ_g ratio is 0.9 and dry K/ µ ratio is about 0.7. For K_g/μ_g ratio of 2, the dry K/µ ratio increases to 0.75. For sands, dry K/µ ratio is not sensitive to mineral properties, which is consistent with data for different mineral sands (Spencer et al., 1994). The Hertz-Mindlin model suggests that dry shear modulus is always greater than the dry bulk modulus. However, we found that for loose sands, the dry bulk modulus seems to be greater than the shear modulus, especially at low pressures.
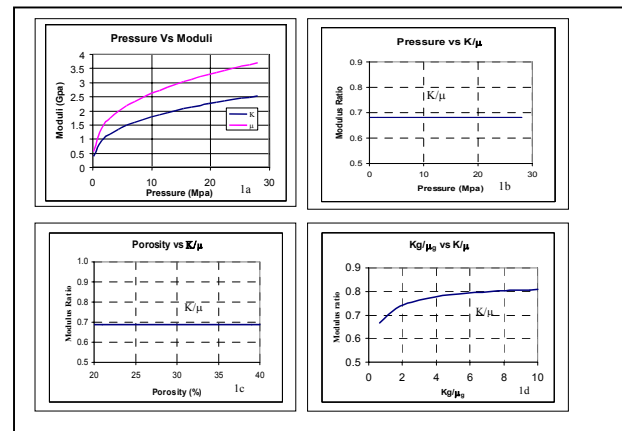
Figure 1. The Hertz-Mindlin (1949) model for loose sands.

The Walton Model (1987) has considered the smoothness of grain contact. Roughness of grain contact can cause a significant increase of dry shear modulus. The initial coherence force or weak cement among grains seems equivalent to the increase in roughness of grain contact, which does not have much effect on the bulk modulus, but has significant effect on the shear modulus. Therefore, for loose sands, the dry K/μ ratio can be widely varied in range from 0.7 to over 1.6 as shown in Figure 2.
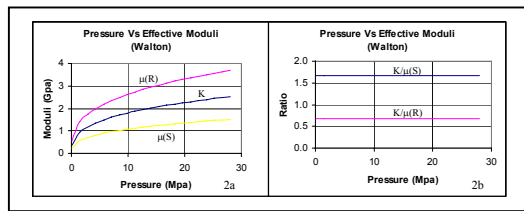

Figure 2. The Melton model (1987) for loose sands. Shear modulus depends on smoothness of grain contact but no effect on bulk modulus (2a). K/μ ratio increases with decreasing roughness (2b).

Grain contact models suggest that dry K/μ ratio remains a constant with varying porosity, differential pressure and mineral properties and vary with roughness of grain contact. Following, we examine measured data to see how the model works.

Dry bulk and shear modulus
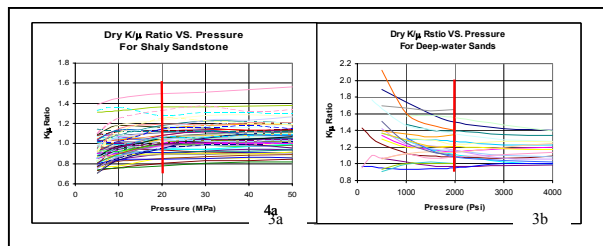

Figure 3. Dry bulk (K_d) and shear (μ) modulus ratio for consolidated rock (3a) and deep-water sands (3b) from Gulf of Mexico as function of differential pressure.

For consolidated sandstone (Han, 1986), the ratio of dry bulk and shear modulus of individual sample tend to be a constant at high pressure (> 20 Mpa) and decreases slightly with decreasing pressure as shown in Figure 3a.

For consolidated sandstone samples, dry K/μ ratio ranges mainly from 0.8 to 1.2. For clean quartz sands, K_d/μ ratio tends to be less than 1. For shaly sands, high K_d/μ ratio is correlated to high porosity and high content of other minerals than quartz in rock frame, such as calcite, feldspar, hematite and so on (Han, 1986). These minerals have higher bulk modulus and lower shear modulus than quartz.

Recently, we have measured velocities on sand samples from the deep-water, Gulf of Mexico. The ratio of dry bulk and shear modulus of individual sample tends to be a constant at a high pressure and increases with decreasing pressure as shown in Figure 3b. Opposite trend of dry K/μ ratio versus differential pressure between consolidated sandstones (Fig. 3a) and loose sands (Fig. 3b) is mainly due to highly pressure dependence of rigidity than bulk modulus of deep-water loose sands. For loose sands, dry bulk modulus K is greater than shear modulus μ and dry K/μ ratio ranges from 1.0 to 1.4. Relatively high K/μ ratio is mainly due to different grain contact (grain texture). Large grain sands seem to have smooth surface as compared to rough (angular) surface of fine sands and cause relatively high K/μ ratio as predicted by Walton model (1987).

Measured data on sands and sandstones suggest that dry K/μ ratio tends to be a constant for a sample and vary systematically with rock texture for different samples. There is intrinsic relation between bulk and shear modulus

Dry bulk and shear modulus relation

It has been suggested that the V_p/V_s ratio of clean sandstone is a constant of 1.5 (Murphy et. al, 1991), which is approximately equivalent to dry K/μ ratio of 0.9. Krief et al. (1990) have proposed that K/μ ratio of dry frame equal to that of mineral. Here we examine data measured on clean and shaly sandstone and weakly cemented sands. The bulk and shear modulus has shown a remarkable correlation as seen in Figure 4.
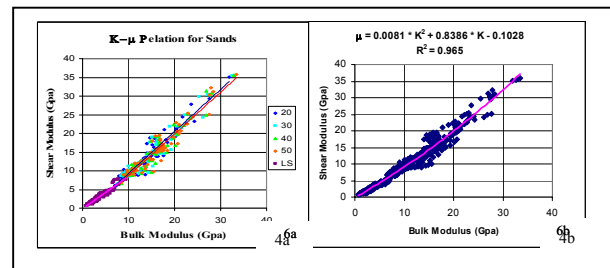
Figure 4. K and μ modulus relation for sandstone and loose sands. (4a) shows data measured at different pressures and fitted model. (4b) shows all the data with unified model.

With sedimentary compaction and cementation, loose sands are gradually consolidated. The data show bulk and shear modulus tends to approach mineral properties along a tight K-μ relation (Fig. 4). There is a slight pressure effect, which is not significant. We fit all the data with polynomial relation:

$$\mu = 0.0081 * K^2 + 0.8386 * K - 0.1028. \qquad (4)$$

The equation shows a simple K-μ relationship for both loose sands and consolidated sandstone. Clearly there is scattering in data, which may be related to the different degree of cementation, different mineral properties, and different grain size and contact.

In practice, P-wave velocity can be measured in seismic data. P-modulus contains a part of shear modulus as shown

$$M = \rho * V_p^2 = K + 4/3 * \mu. \qquad (5)$$

Therefore, shear modulus should have better correlation with dry P-wave modulus as shown in Figure 5.
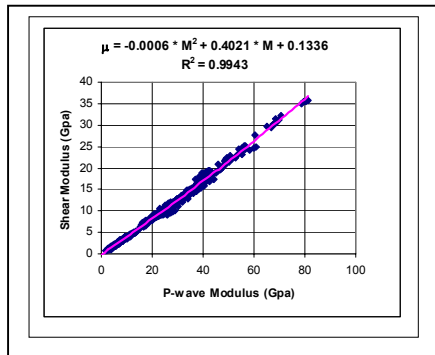

Figure 5. Shear and bulk modulus relation for sandstone and weakly cemented sands.

$$\mu = -0.0006 * M^2 + 0.4021 * M + 0.1336 \qquad (6)$$

with correlation coefficient of 0.99. This improved correlation may suggest that the shear modulus μ is dominated in P-wave modulus. We can use equation (5) to calculated shear modulus directly.

An important issue is that the dry bulk and shear modulus of sandstone are tightly correlated in a simple relationship with distribution of porosity, mineral composition, clay content, cementation, and differential pressure. We can reasonably estimate the shear modulus if we know the P-wave modulus.

Estimating shear modulus using Gassmann's Equation (G-method)

Estimation of shear velocity or examination of quality of measured shear data has been major effort in hydrocarbon evaluation. We propose following method to estimate shear modulus using the above empirical K-μ relationship and approximate Gassmann's equation. Here, we give a step by a step approach:

1. Get consistent rock parameters: P-wave velocity, porosity, and mineral properties of sands, fluid modulus and density. For multiphase fluids, we need to know properties and fractions of each component.

2. Use low frequency approximation of the Gassmann's equation with the P-wave modulus M to calculate fluid saturation effect

$$\Delta K_s = \frac{M_0 * (1 - M_s / M_0)^2}{-1 - \phi + M_s / M_0 + \phi * M_0 / M_f}. \qquad (7)$$

   Equation 7 should be calibrated to understand distribution of the systematic errors.

3. Calculate dry P-wave modulus, which is equal to the difference between M_s and ΔK_s.

4. Calculate shear modulus based on empirical relations for dry P-wave and shear modulus, as in equation (6).

5. Examine the derived dry P-wave modulus (M_d) and shear modulus. The data should be calibrated with local deposition environment, rock and fluid property trend, and general bound for porous media.

Shear velocity can be calculated using this shear modulus and the proper density. Estimated shear velocity with G-method is fairly consistent to those with the methods by Greenberg and Castagna, (1992) and by Xu, and White, (1996) as shown in Figure 6.

Discussion

Predicting shear velocity is a challenge because of complications in rock texture, lithology and pore fluid and reservoir conditions. The method we have proposed can be

widely applied for sands and sandstones. It also provides intrinsic constrain to both P- and S-wave velocity data.

To examine quality of velocity data we suggest applying our derivation to obtain dry bulk modulus. The dry bulk modulus should be consistent with shear modulus as follows:

1. The derived dry bulk modulus must be positive and greater than 2/3 of shear modulus to insure a positive lame constant and Poisson's ratio. Otherwise, either V_p is too low or Vs is too high.

2. Dry K/µ ratio should be consistent with rock parameters, such as it tends to be greater than 1 for loose sands and decreases with increasing differential pressure, and around 1 for consolidated sandstone and increases with increasing porosity, differential pressure and other than quartz minerals (e.g. calcite, feldspar…) .

3. Gassmann's equation is based on a zero frequency assumption to offer a low bound of fluid saturation effect (Han and Batzle, 2002). If P-wave data is in a high frequency domain, the Gassmann's calculation will overestimate dry bulk and shear modulus and velocity. Lower porosity causes a low estimate of dry modulus. Wrong fluid information causes wrong estimate of dry modulus.

Conclusion

Dry P-wave and shear modulus has an intrinsic correlation. These correlations offer a quality control for seismic velocities and interpretation. It also provides a new method to estimate shear velocity based on P-wave modulus and Gassmann's equation.

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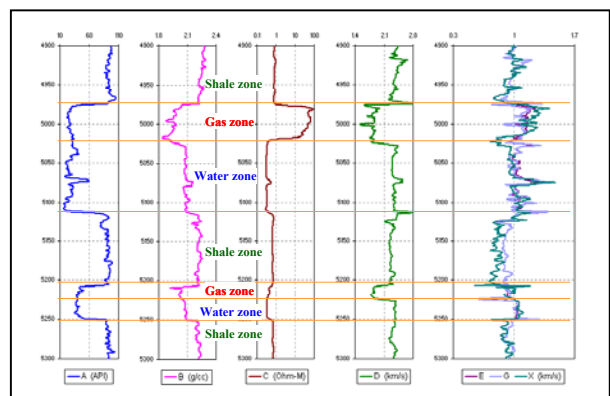
Acknowledgments

Figure 6. Comparison of estimated shear velocities with G-method (G), Greenberg-Castagna method (E) (1992) and Xu-white method (X) (1996).