Correlations between the seismic anisotropy parameters for shales

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Summary

Although the five elastic constants of a transversely isotropic (TI) medium are theoretically independent, correlations among the TI elastic constants exist for natural rocks. Study of the relations between the TI elastic constants is important for practical applications of seismic anisotropy. For shales, c_{13} is practically constrained by c_{11}, c_{33} and c_{66}. The practical bounds on c_{13} are used to evaluate the data quality control of laboratory velocity anisotropy measurements. Excluding data of poor quality control reveals strong correlations between c_{13} and the other TI parameters. A strong correlation is found to exist between δ and the other Thomsen parameters when high-quality datasets are used.

Introduction

Shales or mudstones account for most of the bulk volume of the sedimentary rocks and are the primary factor of seismic anisotropy in seismic exploration (Schoenberg et al., 1996). The elastic properties of shales are often described by transverse isotropy (Vernik and Nur, 1992; Johnston, 1995; Vernik and Liu, 1997; Jakobsen and Johansen, 2000; Sondergeld et al., 2000; Wang, 2002; Sondergeld and Rai, 2011; Sone, 2012). The challenges of including anisotropy in seismic-data processing and interpretation come from introduction of extra parameters. For isotropic rocks, although the elastic properties are defined by two independent parameters (V_P and V_S), there are often good correlations between them (Castagna et al., 1985). These correlations play an important role in seismic exploration. For the most simple and practical case of transverse isotropy, the elastic properties are defined by five independent elastic parameters. Similar to the isotropic case, if we could find connections among the five independent elastic parameters, it may greatly simplify the problems in anisotropic seismic data processing and interpretation.

Theory

The elastic properties of a TI medium are specified by five independent elastic constants (c_{11}, c_{33}, c_{44}, c_{66}, and c_{13}). The concept of Poisson's ratio for an isotropic medium can be straightforwardly extended to a TI medium using Hook's law (King, 1964; Yan et al., 2016). Their relations with the TI elastic constants are as follows

$$\nu_V = \frac{c_{13}}{2(c_{11}-c_{66})} \quad (= \nu_{31} = \nu_{32}), \tag{1}$$

$$\nu_{HV} = \frac{2c_{13}c_{66}}{c_{11}c_{33}-c_{13}^2} \quad (= \nu_{13} = \nu_{23}), \tag{2}$$

$$\nu_{HH} = \frac{c_{33}(c_{11}-2c_{66})-c_{13}^2}{c_{11}c_{33}-c_{13}^2} \quad (= \nu_{12} = \nu_{21}). \tag{3}$$

The coordinate system used for the notation is shown in Figure 1. Figure 1 also shows the diagram of the deformation of a horizontal plug under uniform axial compression. The deformation in the radial directions of the cylindrical sample will not be uniform due to elastic anisotropy, and two principal Poisson's ratios (ν_{HH} and ν_{HV}) can be measured from the compressional testing. Based on static mechanic measurement and physical intuition, Yan et al. (2016) argued that a certain relationship exists practically between these two principal Poisson's ratios for hydrocarbon source rocks:

$$0 < \nu_{HH} < \nu_{HV}. \tag{4}$$

From these relations they derived tight constraints on c_{13},

$$c_{13}^- < c_{13} < c_{13}^+, \tag{5}$$

where

$$c_{13}^- = \sqrt{c_{33}(c_{11}-2c_{66}) + c_{66}^2} - c_{66},$$

$$c_{13}^+ = \sqrt{c_{33}(c_{11}-2c_{66})}.$$

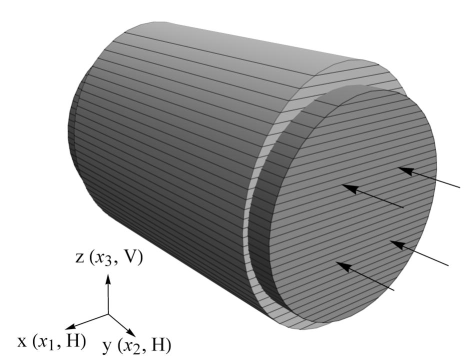

Figure 1: Diagram of deformation of a horizontal core plug under uniform axial compressional stress and the coordinate system.

Laboratory velocity anisotropy measurement

Experimental determination of the four elastic constants defining the elastic properties in non-oblique directions are straightforward from four velocity measurements: $c_{11} = \rho V_{P90}^2$, $c_{33} = \rho V_{P0}^2$, $c_{44} = \rho V_{SV0}^2$ and $c_{66} = \rho V_{SH90}^2$. Here the subscripts P, SV and SH denote the three wave modes in an anisotropic medium, and the numbers in the subscripts denote directions in degrees with respect to the TI symmetric axis. At least one quasi-P wave or SV-wave velocity must be made in an oblique direction to determine c_{13} (Yan et al., 2012):

$$c_{13} = 2 \csc 2\theta \sqrt{D} - c_{44} \tag{6}$$

where

$$D = (\rho V_{P\theta}^2 - c_{11} sin^2\theta - c_{44} cos^2\theta)(\rho V_{P\theta}^2 - c_{33} cos^2\theta - c_{44} sin^2\theta)$$

where the subscript θ denotes phase angle. In an anisotropic medium, the energy propagation direction may be different from the instantaneous direction of particle motion. There are differences between the group velocity and phase velocity, and group angle and phase angle. If an oblique group velocity is measured, c_{13} is can be numerically computed using equation (6) and the following relations (Byun, 1984).

$$\mathrm{Tan}(\varphi - \theta) = \frac{1}{V_\theta}\frac{\mathrm{d}V_\theta}{\mathrm{d}\theta}, \tag{7}$$

$$V_\theta = V_\varphi \cos(\varphi - \theta) . \tag{8}$$

where V can be either P, SV or SH wave velocity, φ is the group angle and denotes the group velocity when used as a subscript.

Experimental data analysis

Yan et al. (2013; 2014; 2016) analyzed various factors that may cause significant uncertainty in laboratory velocity anisotropy measurement. Genuine phase velocity measurement requires the piezoelectric transducer is sufficiently wide to receive the deviated plane wave, at the same time the sample length should not be too long. Genuine group velocity measurement requires energy emitted from a point source be received by a point receiver in the cross-section passing the TI symmetrical axis. These delicacies are often overlooked by previous experimental studies and lead to significant uncertainties in experimental determination of c_{13} (Yan, et al., 2014; Yan, 2015).

The data quality of laboratory velocity anisotropy measurement may be evaluated using the practical bounds on c_{13} by equation (5). Figure 2 shows the crossplot between c_{13} and the ν_{HH}/ν_{HV} ratio from velocity anisotropy measurement. The data sources are from Thomsen (1986), Johnston and Christensen (1995), Vernik and Liu (1997), Jakobsen and Johansen (2000), Wang (2002b, shale and coal samples only), and Sone (2012). The crossplot can be divided into three areas. In the left, several data points have negative ν_{HH} values, which physically mean that the horizontal plugs under uniform axial compression will shrink along the bedding direction. The corresponding c_{13} values are above the high bound. In the right area, there are quite a few points with $\nu_{HH}>\nu_{HV}$, which physically mean that

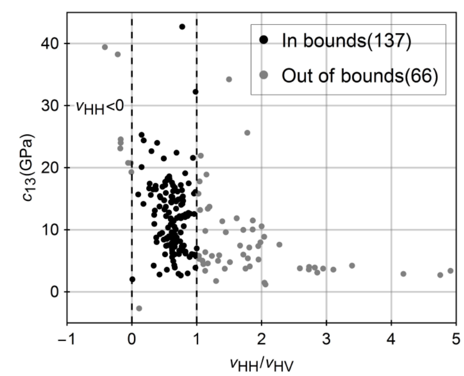

Figure 2: Correlation between c_{13} and the ratio of Poisson's ratios. The dark data points have c_{13} value within its practical bounds and the gray data points have c_{13} value out of its practical bounds.

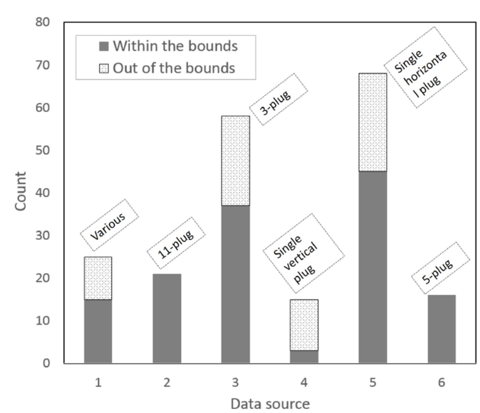

Figure 3: Data quality evaluation of the data sources using the practical bounds on c13. Count denotes the number of data points. Different measurement methods are annotated for each data source. The data sources come from: 1. Thomsen, 1986; 2. Johnston and Christensen, 1995; 3. Vernik and Liu, 1997; 4. Jakobsen and Johansen, 2000; 5. Wang, 2002b; 6. Sone, 2012.

the horizontal plugs under uniform axial compression will expand more along the bedding than in direction perpendicular the bedding. It is well known that the shales are stiffer along the bedding than in direction perpendicular to the bedding, the data points in the right section have underestimated c_{13} values. About 2/3 of the data points lay in the center area defined by the practical bounds of c_{13}. There is one data point with negative c_{13} in the central area. The data point has negative ν_V and it may be caused by experimental error. Therefore, in terms of c_{13} determination, there is significant uncertainty in the experimental anisotropy studies. Of course, if the sample has significant heterogeneity and it cannot be treated as an effective TI medium, the estimated c_{13} value may be out of its practical bounds.

Figure 3 shows the statistics of the data points with c_{13} in the bounds or out of the bounds for each data source used in Figure 2. It can be seen that the estimated c_{13} values all lie within its practical bounds fore data sources 2 and 6. They have a common feature: multiple oblique velocities are measured to estimate c_{13} by least square regression. Each data point in dataset 2 is based on measurement of 11 core plugs in different directions, and each data point in dataset 6 is based measurement of 5 core plugs in different directions (0^o, 30^o, 45^o, 60^o, 90^o). Data source 1 (Thomsen, 1986) is collected from various sources, the measurement was conducted before discussion of phase and group velocity confusion in the oblique velocity measurement by Dellinger and Vernik (1994). It is not a surprise that the data quality control of data source 1 is not good. In data source 3, P-wave piezoelectric transducer of 12 mm diameter is used for phase velocity measurement on a 45^o plug. The piezoelectric transducer may not be sufficiently wide for some strongly anisotropic samples. For data source 4, all the measurements are based on a single vertical plug, and the setup is defective for genuine group velocity measurement in the oblique direction. For data source 5 (Wang 2002b), all the measurements are conducted on a single horizontal plug, and the group velocity is mistaking as phase velocity in the original data (Yan et al., 2016).

In summary, multiple oblique velocity measurement is critical for reduction of uncertainty in estimation of c_{13}, and data sources 2 and 6 have better quality control than the other data sources in the determination of c_{13}.

Correlations among the TI elastic constants

There are usually strong correlations between the elastic constants determining the elastic properties in the non-oblique directions (Horne, 2013), but the mutual relations between c_{13} and the other TI elastic constants are not clear. Instead, we use multivariable regression to correlate c_{13} with the other TI elastic constants together. As shown in the top

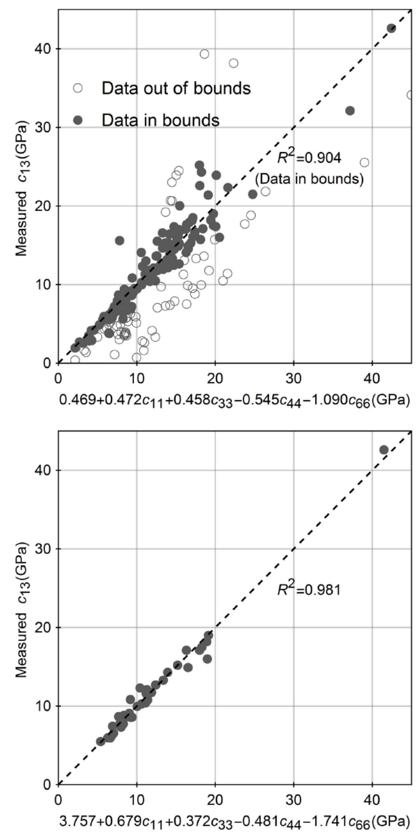

Figure 4: Correlation between c_{13} and the other TI elastic constants using data of different quality control. The correlation in the top panel is based on the data points with c_{13} inside its practical bounds and the correlation in the bottom panel is based on data sources 2 and 6 as shown in Fig. 3.

panel of Figure 4, the correlation between c_{13} and the other TI elastic constants is strong when only data points with c_{13} lying within the practical bounds are used. If all the data points are used, the R^2 correlation coefficient is reduced to 0.751 from 0.904. It is noticed that most of the data points with c_{13} lying out of the practical bounds are distributed further away from the regression trend.

As shown in Figure 3, if a dataset has a lot of data points with c_{13} values lying out of the bounds, then the data points with c_{13} values lying in the bounds may still have significant uncertainty in the estimation of c_{13}. It is logical to believe that the data subset excluded of data points out of the bounds is still less reliable than the other dataset originally with all the c_{13} lying within the bounds. Therefore, if we use only data sources 2 and 6, the correlation should be further improved if the strong correlation intrinsically exists. As

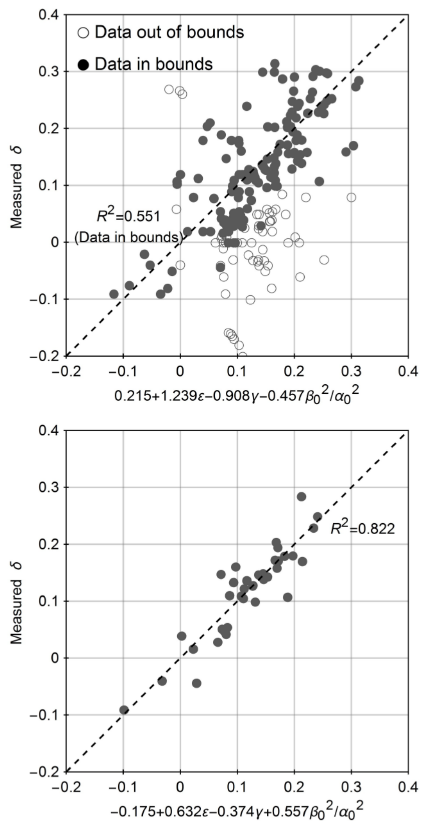

Figure 5: Correlation between δ and the other Thomsen parameters. The correlation in the top panel is based on the data points with c_{13} inside its practical bounds and the correlation in the bottom panel is based on data sources 2 and 6 as shown in Fig. 3.

seen from the bottom panel in Figure 4, indeed the correlation is significantly improved. This means that for shales, c_{13} is not an independent parameter and it is generally determined by the other TI elastic constants. Since c_{13} is usually much more difficult to be obtained than other TI elastic constants, this correlation may be quite useful in practical applications. Although the number of data points is significantly reduced, the shale samples used in datasets 2 and 6 come from eight different areas and formations all over the world. This correlation is still representative in a certain degree.

Correlation between the Thomsen parameters

Reliable estimation of the anisotropy parameters is the foundation for anisotropic seismic data processing and interpretation. If a good relation between δ and the other anisotropy parameters can be found, we may mathematically simplify the problem or use it as a constraint for anisotropic parameter estimation. Of the three Thomsen parameters, δ is most difficult to be reliably determined from laboratory measurements and its physical meaning of δ is not clear, but it is the most important parameter in anisotropic seismic data processing because it determines the relation between the vertical velocity and the normal move out velocity (Thomsen, 1986).

Figure 5 shows the correlations between δ and the others Thomsen parameters using all the data sources (top panel) and data sources 2 and 6 only (bottom panel). The correlation coefficient is 0.207 when all the data points are used and 0.551 when only data points with c_{13} lying in the bounds are used. If only data sources 2 and 6 are used, the correlation coefficient is improved to 0.822. The steady improvement of the correlation when laboratory measurements data of better quality control are used shows the rationality of using the practical constraints of c_{13} as a tool of quality control for laboratory velocity anisotropy measurements.

Conclusions

In laboratory velocity anisotropy measurement, there are significant uncertainties in the determination of c_{13} and Thomsen parameters δ. The uncertainty can be significantly reduced if velocities in multiple oblique directions are measured. The practical constraints on c_{13} can be a tool of quality control for velocity anisotropy measurement. Using the data sources with good quality control, it is found that strong correlations exit between c_{13} and the other TI elastic, and between the Thomson parameter δ and the other Thomsen parameters.

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EDITED REFERENCES

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