A study of Multiscale Seismic Data Joint Inversion method

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Summary

Most conventional inversion methods are mainly based on surface seismic data. Although it has good lateral continuity, its vertical resolution is poor, due to the limitations of surface seismic data. In this paper, we integrate the surface seismic data, VSP data, and crosswell data in the inversion equation. Because these different data have different advantages, they can compensate for each other, and thus improve the inversion resolution. Also, we used the modified Cauchy prior information of reflectivity, and made the statistics of well data, which demonstrated its reliability. To solve the inversion equation, I adopted the modified PRP conjugate gradient method, which is stable and faster than matrix inverse method. Finally, we tested this inversion method with 2D model and real survey data, which suggested this method has better precision.

Introduction

Multiscale seismic data joint inversion is proposed by DanPing Cao in 2009. In this method he integrated the surface seismic data, VSP data, and crosswell data in the inversion equation (Danping et al., 2009). Although he adopted the modified Cauchy prior information as constraint of reflectivity, he only showed the superiority of this prior distribution theoretically, without proving whether this distribution conforms to real seismic data or not. In this paper, I show the statistical histogram of real well data, demonstrating that the modified Cauchy distribution conforms the real well data well. Besides, the influence of the parameters on the inversion result is studied. Since this inversion method involves many seismic data, it is important to improve the solving method, so as to save time. I modified the PRP conjugate gradient method, mainly by changing the direction parameter, made it search the direction faster, thus reduced the iteration times, and increased the convergent rate.

Method

Based on the convolution model:

$$d = Gm + n \tag{1}$$

where D is data vector, G is kernel, m is model parameter, and n is noise.

Conventional inversion method tried to minimize the noise, but since prior information about model parameter is introduced, we minimize both the noise and norm of the solution here, as shown in equation (2) (Tarantola, 2005):

$$\min[\| d - Gm \|_2 + \lambda \| m \|_1] \tag{2}$$

Bayesian Theory

According to Bayesian Inference Theory, once the prior probability and the likelihood function is known, the posterior probability can be obtained by multiplying them, like:

$$p(r \mid d) = \frac{h(d,r)}{p(d)} = \frac{p(r)p(d \mid r)}{\int p(r)p(d \mid r)dr} \propto p(r)p(d \mid r) \tag{3}$$

where p(r) is the prior probability of random variable r, p(d|r) is the likelihood function, and p(r|d) is the posterior probability (Ulrych T et al., 2001). Figure 1 shows their relationship.

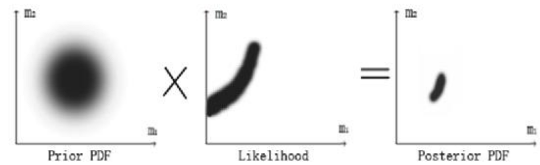

Figure 1. The Bayesian inference theory

Once the posterior probability is obtained, according to Maximum Posterior Probability (MAP), the optimal value can be solved by maximize the posterior probability. MAP states that the random variable is at its optimal value when the MAP reaches the maximum value.

Assume the noise in surface seismic data obeys Gaussian distribution, of which the mean value is 0, and variance is σ_n^2. Then likelihood function of reflectivity $p(\mathbf{d}_S \mid \mathbf{r})$ can be represented in the form of the noise distribution, as:

$$P(d_S \mid r) = P(n_S) = \frac{1}{(2\pi\sigma_n^2)^{N/2}} \exp\left\{ \frac{-(d_S - G_S r)^T (d_S - G_S r)}{2\sigma_n^2} \right\} \tag{4}$$

Where d_S is the surface seismic data, n_S is noise, G_s is the surface wavelet matrix, σ_n is the standard variance of noise.

Similarly, assume the noise in VSP data and crosswell data also obeys Gaussian distribution, then we have:

$$P(d_v \mid r) = P(n_v) = \frac{1}{(2\pi\sigma_v^2)^{N/2}} \exp\left\{ \frac{-(d_v - G_v r)^T (d_v - G_v r)}{2\sigma_v^2} \right\} \tag{5}$$

$$P(d_w \mid r) = P(n_w) = \frac{1}{(2\pi\sigma_w^2)^{N/2}} \exp\left\{ \frac{-(d_w - G_w r)^T (d_w - G_w r)}{2\sigma_w^2} \right\} \tag{6}$$

where d_v is the VSP data, d_w is the crosswell data, n_v is noise of VSP, n_w is noise of crosswell, G_v is the VSP wavelet matrix, G_w is the crosswell wavelet matrix, σ_v is the standard variance of VSP noise, σ_w is the standard variance of crosswell noise.

Surface seismic data, VSP data and crosswell data reflect the characteristics of the same geologic body, namely, they are the responses of the same geologic body, but they are acquired and processed independently. Therefore, the joint

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likelihood function of them can be calculated by multiplying the independent likelihood functions (Laurence, 1988), as shown in equation (7):

$$p(\boldsymbol{d} \mid \boldsymbol{r}) = K_1 \cdot \exp\left\{\frac{-(d_S - G_S r)^T (d_S - G_S r)}{2\sigma_n^{\ 2}}\right\} \cdot \exp\left\{\frac{-(d_w - G_w r)^T (d_w - G_w r)}{2\sigma_w^{\ 2}}\right\} \cdot \exp\left\{\frac{-(d_v - G_v r)^T (d_v - G_v r)}{2\sigma_v^{\ 2}}\right\} \quad (7)$$

where , $K_1 = \dfrac{1}{(2\pi)^{3N/2} \left|\sigma_n^{\ 2}\sigma_w^{\ 2}\sigma_v^{\ 2}\right|^{1/2}}$

Further, assume the reflectivity obeys Gaussian distribution, of which the mean value is r , and variance is $\sigma_r^{\ 2}$, then we have the prior probability of reflectivity, as:

$$p(\boldsymbol{r}) = \frac{1}{(2\pi\sigma_r^{\ 2})^{M/2}} \exp\left(\frac{-r^T r}{2\sigma_r^{\ 2}}\right) = K_2 \exp\left(\frac{-r^T r}{2\sigma_r^{\ 2}}\right) \quad (8)$$

Where, $K_2 = \dfrac{1}{(2\pi\sigma_r^{\ 2})^{M/2}}$

Then according to Bayesian Inference Theory, the posterior probability can be calculated as:

$$\begin{aligned} p(\mathbf{r} \mid \mathbf{d}) &\propto p(\mathbf{r}) \cdot p(\mathbf{d} \mid \mathbf{r}) \\ &= K_1 K_2 \exp\left(\frac{-r^T r}{2\sigma_r^{\ 2}}\right) \cdot \exp\left\{\frac{-(d_S - G_S r)^T (d_S - G_S r)}{2\sigma_n^{\ 2}}\right\} \cdot \\ &\quad \exp\left\{\frac{-(d_w - G_w r)^T (d_w - G_w r)}{2\sigma_w^{\ 2}}\right\} \cdot \exp\left\{\frac{-(d_v - G_v r)^T (d_v - G_v r)}{2\sigma_v^{\ 2}}\right\} \\ &= K_1 K_2 \exp\left\{\frac{-r^T r}{2\sigma_r^{\ 2}} - \frac{(d_S - G_S r)^T (d_S - G_S r)}{2\sigma_n^{\ 2}} - \frac{(d_w - G_w r)^T (d_w - G_w r)}{2\sigma_w^{\ 2}} - \frac{(d_v - G_v r)^T (d_v - G_v r)}{2\sigma_v^{\ 2}}\right\} \end{aligned} \quad (9)$$

According to MAP, the optimal reflectivity can be obtained by maximizing the posterior probability. Simplify equation (9), and introduce the impedance constraint (Paul Glederblom, Jaap Leguijt, 2010), we have the final objective function as:

$$\begin{aligned} J(r) &= J_S + \alpha J_V + \beta J_C + \mu J_r + \rho J_I \\ &= \frac{1}{2}(d_S - G_S r)^T (d_S - G_S r) + \frac{\alpha}{2}(d_V - G_V r)^T (d_V - G_V r) \\ &\quad + \frac{\beta}{2}(d_C - G_C r)^T (d_C - G_C r) + \frac{\mu}{2} r^T r + \frac{\rho}{2}(Cr - \xi)^T (Cr - \xi) \end{aligned} \quad (10)$$

Prior distribution of reflectivity

Commonly used prior distributions are (Danilo, 2008):
Gaussian distribution:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad (11)$$

Cauchy distribution:

$$f(x; x_0, \gamma) = \frac{1}{\pi}\left[\frac{\gamma}{(x - x_0)^2 + \gamma^2}\right] \quad (12)$$

Here we use the modified Cauchy distribution:

$$f(x; x_0, \gamma) = \frac{1}{\pi\gamma} e^{\frac{-(x-x_0)^2}{\gamma^2 + (x-x_0)^2}} \quad (13)$$

Figure 2 shows the comparison of these different distributions.

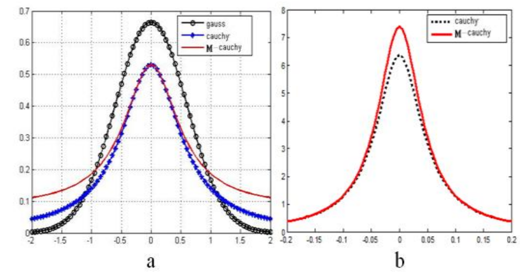

Figure 2. Comparison of different distributions

Figure 2(a) suggests that modified Cauchy has long tail distribution characteristic, which can help realize the sparseness of the reflectivity; Figure 2(b) suggest that around zero modified Cauchy has higher value than Cauchy distribution, indicating that the modified Cauchy can better protect weak reflectivity.

To test the agreement of modified Cauchy distribution with real data, the statistical simulation of four wells is shown in Figure 3.

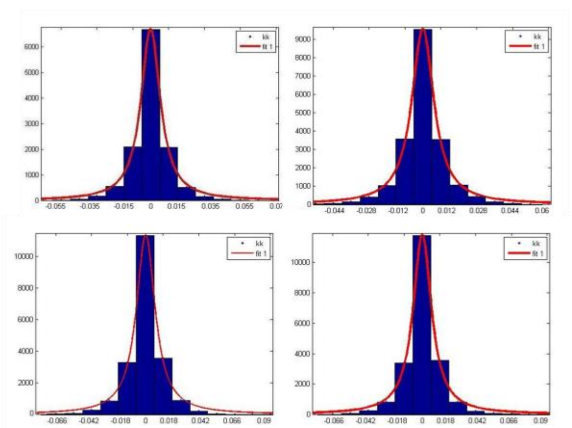

Figure 3. Statistical histogram of well data and simulation of modified Cauchy distribution simulation

Through Figure 3, it can be seen that modified Cauchy distribution fits the real well data well, which demonstrates the rationality and reliability of modified Cauchy distribution. The result of these different prior distributions is shown in Figure 4.

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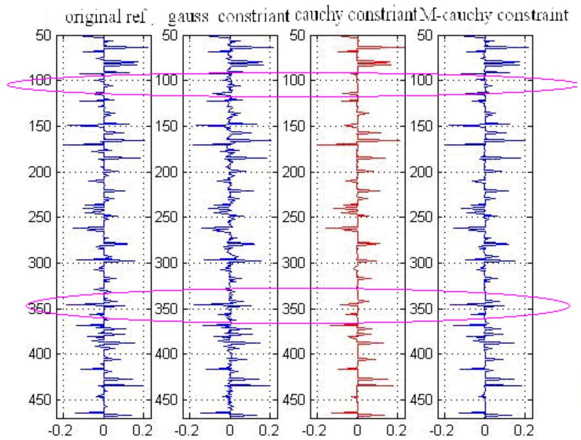

Figure 4. The result of different prior distributions

The correlation coefficient between model reflectivity and gauss distribution result is 0.708, for Cauchy distribution is 0.920, and for modified Cauchy distribution is 0.978. Therefore, the result of modified Cauchy distribution is better, and Figure 4 also demonstrates that modified Cauchy distribution not only realizes the sparseness of the result, but also protects the weak reflection well.

To solve equation 10, modified PRP conjugate gradient method is used. The parameters of this method are:
Iteration equation:

$$r_{k+1} = r_k + \alpha_k d_k, \qquad k = 0,1,\ldots \tag{14}$$

Search direction:

$$d_k = \begin{cases} -g_k, & if \quad k = 0 \\ -g_k + \beta_k d_k - \dfrac{g_k^T d_{k-1}}{\| g_{k-1} \|^2}(g_k - g_{k-1}), & if \quad k \ge 1 \end{cases} \tag{15}$$

Direction parameter:

$$\beta_k = \frac{g_{k+1}^T (g_{k+1} - g_k)}{g_k^T g_k} \tag{16}$$

Figure 5 shows the comparison of modified PRP method with conventional matrix inverse method.

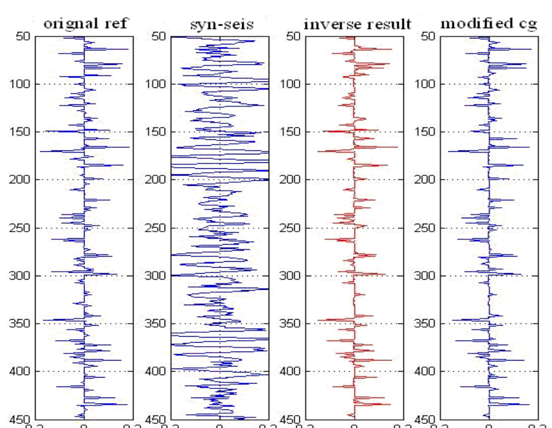

Figure 5. Comparison of modified cg and matrix inverse method

The result of the modified PRP cg is almost the same as the result of matrix inverse method, which demonstrates the stability and reliability of this modified method. However, this method has faster convergent rate than conventional method, as shown in Figure 6.

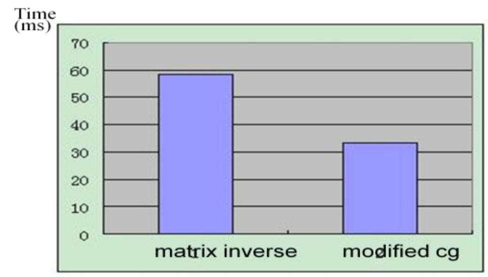

Figure 6. Run time comparison of modified cg and matrix inverse method

Figure 6 suggests that this modified PRP cg method uses much less time to solve the equation, which indicates that modified method has faster convergent rate.

To test the influence of the parameter, we changed the parameter, and show the corresponding result in Figure 7, 8, and 9. The parameter lambda is the weight coefficient of prior constraint in the final equation, and delta is the parameter in prior constraint, it is related to the reflectivity variance.

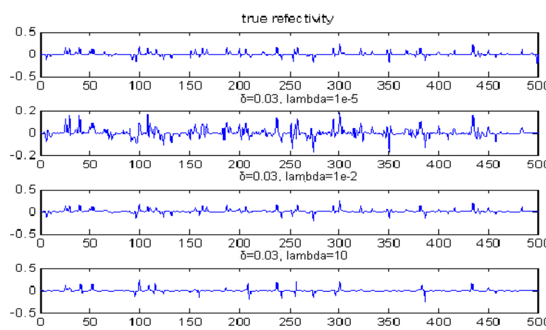

Figure 7. The influence of lambda on result

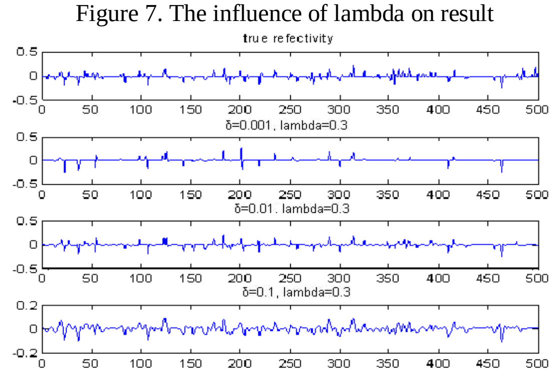

Figure 8. The influence of delta on result

Figure 7 and 8 indicate that delta and lambda, work the opposite way. When Lambda increases, the result becomes sparser; when delta increases, the result becomes denser. Figure 9 shows the influence of delta and lambda on the final objective function.

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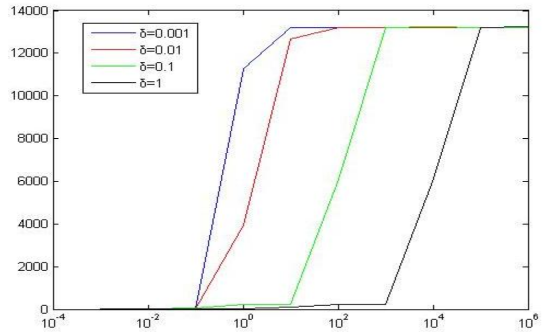

Figure 9. The influence of lambda and delta on objective function value

Through Figure 9, we can see that when delta increases, the objective function value decreases; when lambda increases, the function value increases.

Example

Figure 11 shows the test of this inversion method with 2D model. The left one is original model, the middle one is conventional inversion result, the right one is the joint inversion method.

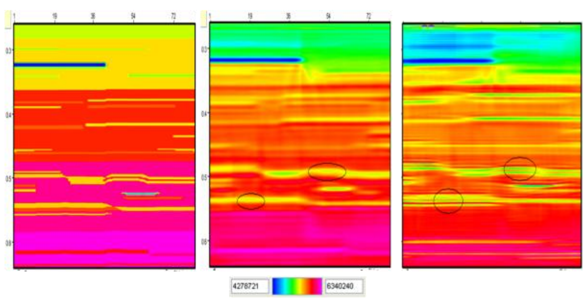

Figure 10. Test with 2D model

Through comparison, we can see that joint inversion method has higher vertical resolution. Figure 12 shows the test with real survey data, the upper one is conventional inversion result, and the below one is the joint inversion result.

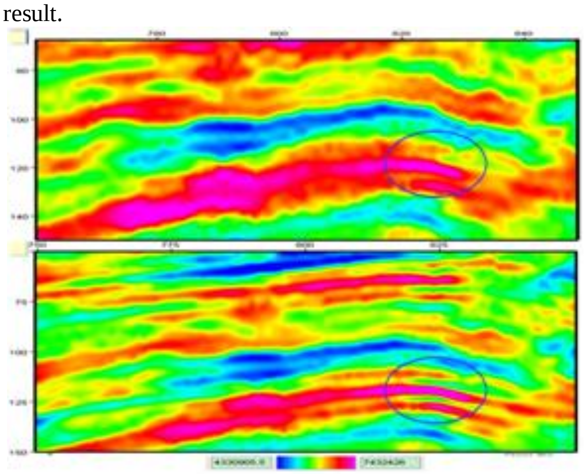

Figure 11. Test with real survey data

In Figure 12, we can easily see that joint inversion method has higher vertical resolution than conventional method, and can identify thin layers that conventional method can't identify.

Conclusions

Through test with model and real survey data, we can see that multiscale seismic data joint inversion can integrate different-scale seismic data, take use of advantages of different data, and thus improve the inversion precision. Compared with commonly used gauss distribution and Cauchy distribution, modified Cauchy prior constraint fits the real data well, and can effectively protect weak reflectivity. Modified PRP conjugate gradient method is a stable method, and it has faster convergent rate than conventional method.

Although the model test and real seismic data test show that multiscale seismic data inversion method is successful in enhancing the resolution, there is a lot work to do. The first one is that: surface seismic data, VSP data, and crosswell data are acquired and processed independently, then to match their phase is a difficult problem. How to make sure that their phase fit each other well is a very basic and important issue. Moreover, even though we have studied the influence of the parameters on inversion result, there are no criteria to make sure that the parameters we choose is appropriate and reliable. Besides, in the inversion equation, we didn't include the factors, such as porosity, permeability, and fluid effect, all of which can influence the result. All in all, plenty of work needs to do to improve the method.

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