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Bayesian Linearized AVAZ Inversion in HTI Fractured Media

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SUMMARY

A new linearized AVAZ inversion method for full elastic properties determination for HTI cracked media in Bayesian framework is developed. The inversion algorithm is based on the convolution model and Ruger's PP reflectivity approximation in HTI media. The objective is to quantify elastic and anisotropic parameters and their associated uncertainty from prestack azimuthal seismic data. Crack density can also be estimated from the inversion results. Tests on synthetic data show that all inverted parameters were almost perfectly retrieved when the noise approached zero, while the three anisotropic parameters are more sensitive to noise level compared to elastic parameters.

Introduction

Vertical fractures often exist in the subsurface and areas of high fracture density may form sweet spots for conventional and unconventional hydrocarbon reservoirs. Such vertically aligned fractures in isotropic matrix are frequently described by transversely isotropic model with a horizontal axis of rotational symmetry (HTI media). Anisotropy serves as a good indicator for fracture density and hence permeability, so obtaining anisotropic information and its associated uncertainty from seismic data is essential for characterizing fractured reservoirs. We cast our AVAZ inversion problem in the Bayesian framework, which is capable of integrating prior information and seismic measurements to quantify uncertainty of the anisotropic parameters.

Methodology

The objective of this inversion is to obtain a posterior distribution for both elastic and anisotropic parameters, which can be related to crack density and its associated uncertainty. Amplitude variation with offset and azimuth (AVAZ) has extensively been used to estimate anisotropic information and crack density through Rüger's P-P reflectivity approximation (1997):

$$R_{pp}(\phi,\theta)=\frac{1}{2}\frac{\Delta Z}{Z}+\frac{1}{2}\left\{\frac{\Delta\alpha}{\bar{\alpha}}-\left(\frac{2\bar{\beta}}{\bar{\alpha}}\right)^{2}\frac{\Delta G}{G}+\left[\Delta\delta^{V}+2\left(\frac{2\bar{\beta}}{\bar{\alpha}}\right)^{2}\Delta\gamma\right]\cos^{2}\phi\right\}\sin^{2}\theta$$

$$+\frac{1}{2}\left\{\frac{\Delta\alpha}{\bar{\alpha}}+\Delta\varepsilon^{V}\cos^{4}\phi+\Delta\delta^{V}\sin^{2}\phi\cos^{2}\phi\right\}\times\sin^{2}\theta\tan^{2}\theta \qquad (1)$$

Where θ and ϕ are the incident and azimuthal angles respectively. α is the vertical P-wave velocity, β is the vertical S-wave velocity, ρ is density, γ is the Thomsen's parameter that is responsible for shear wave splitting. The coefficients of the equivalent VTI model ε^{V} and δ^{V} can be expressed through the generic Thomsen's parameters ε and δ as defined with respect to the horizontal symmetry axis. We rewrite Rüger's PP reflectivity approximation as:

$$R_{pp}(\phi_i,\theta_i)=R_{pp-iso}(\theta_i)+R_{pp-ani}(\phi_i,\theta_i) \qquad (2)$$

Where R_{pp-iso} is based on Aki-Richards's weak contrast approximation to isotropic PP reflectivity, which can be expressed as:

$$R_{pp-iso}(\theta_i)=A(\theta_i)\frac{\Delta\alpha}{\bar{\alpha}}+B(\theta_i)\frac{\Delta\beta}{\bar{\beta}}+C(\theta_i)\frac{\Delta\rho}{\bar{\rho}} \qquad (3)$$

And $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\rho}$ are the averages over reflecting interface; $\Delta\alpha$, $\Delta\beta$, $\Delta\rho$ are the corresponding contrast. PP reflectivity's anisotropic contribution can be expressed as:

$$R_{pp-ani}(\phi_i,\theta_i)=D(\phi_i,\theta_i)\Delta\varepsilon^{V}+E(\phi_i,\theta_i)\Delta\gamma+F(\phi_i,\theta_i)\Delta\delta^{V} \qquad (4)$$

The discrete matrix formulation of the azimuth seismic wiggle can be expressed as,

$$d=Gm+e \qquad (5)$$

Where d refers to the seismic data with azimuth and offset, e is the error term, and m is the earth model which can be expressed as logarithm of elastic parameters and anisotropic parameters:

$$m = [\ln\alpha, \ln\beta, \ln\rho, \varepsilon^V, \gamma, \delta^V]^T$$

The matrix G is constructed as a linear operator which can be written as:

$$G = WAD \qquad (6)$$

Where W is a block-diagonal matrix representing wavelet, D is a differential matrix giving the contrast of elastic properties and anisotropic parameter in m, and A is the azimuth and incident angle-dependent matrix defined below:

$$A = \begin{bmatrix} A(\theta_1) & B(\theta_1) & C(\theta_1) & D(\theta_1,\phi_1) & E(\theta_1,\phi_1) & F(\theta_1,\phi_1) \\ A(\theta_2) & B(\theta_2) & C(\theta_2) & D(\theta_2,\phi_2) & E(\theta_2,\phi_2) & F(\theta_2,\phi_2) \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ A(\theta_n) & B(\theta_n) & C(\theta_{n\theta}) & D(\theta_n,\phi_n) & E(\theta_n,\phi_n) & F(\theta_n,\phi_n) \end{bmatrix}$$

For linear inversion problems, the expectation and covariance of the posterior distribution can be described by the following analytical expressions (Tarantola, 1987, Buland and Omre, 2003):

$$\mu_{m|d} = \mu_m + \sum\nolimits_m G^T (G \sum\nolimits_m G^T + \sum\nolimits_e)^{-1} (d - G\mu_m), \qquad (7\text{-}1)$$

$$\sum\nolimits_{m|d} = \sum\nolimits_m - \sum\nolimits_m G^T (G \sum\nolimits_m G^T + \sum\nolimits_e)^{-1} G \sum\nolimits_m . \qquad (7\text{-}2)$$

The input for this inversion method would be a true-amplitude processed prestack seismic data with azimuthal coverage. The prior model for the inversion is constructed by computing the expectation μ_m and covariance σ_m, which are often estimated from well log data, geo-mechanical model and geologic constraints. We take the second order moments of the spatial dependencies to covariance matrix (Buland and Omre, 2003).

Synthetic AVAZ inversion result

We constructed a synthetic earth profile shown in Figure 1. The elastic parameters, P-wave velocity, S-wave velocity, density and the anisotropic parameters, $\varepsilon^V, \gamma, \delta^V$ are randomly simulated with a Gaussian distribution. Seismic forward modeling is performed based on the convolutional model introduced above. A synthetic gather with azimuths and incident angles is displayed in Figure 2. The wavelet is a Ricker wavelet with 25Hz center frequency and normalized amplitude. It is clearly seen that the seismic amplitudes vary with different azimuths and incident angles.

We tested the Bayesian AVAZ inversion on a random well log model with different noise levels. The inversion result given the input synthetic seismic and the random well log model with S/N ratio of 100 is shown in Figure 3. The inverted maximum posterior solution (red thick line) agrees well with the model, and the 0.95 confidence region indicates that the inversion uncertainty is kept at a low level. Both the elastic and anisotropic parameters can be effectively estimated from seismic data with high S/N ratio. Crack density can be estimated based on the relationship between the crack density of a fractured medium with aligned cracks, and the inverted shear-wave splitting γ (Bakulin, 2000). The posterior distribution of the inversion result with S/N ratio of 4 is displayed in Figure 4. Good predictions are proportional to the S/N ratio. It is also found that the anisotropic parameters $\varepsilon^V, \gamma, \delta^V$ are more sensitive to S/N ratio than the elastic parameters α, β, ρ. When S/N ratio is 4, elastic parameters can still be retrieved to some degree, while azimuthal seismic data almost provides no practical information for estimating anisotropic parameters $\varepsilon^V, \gamma, \delta^V$.

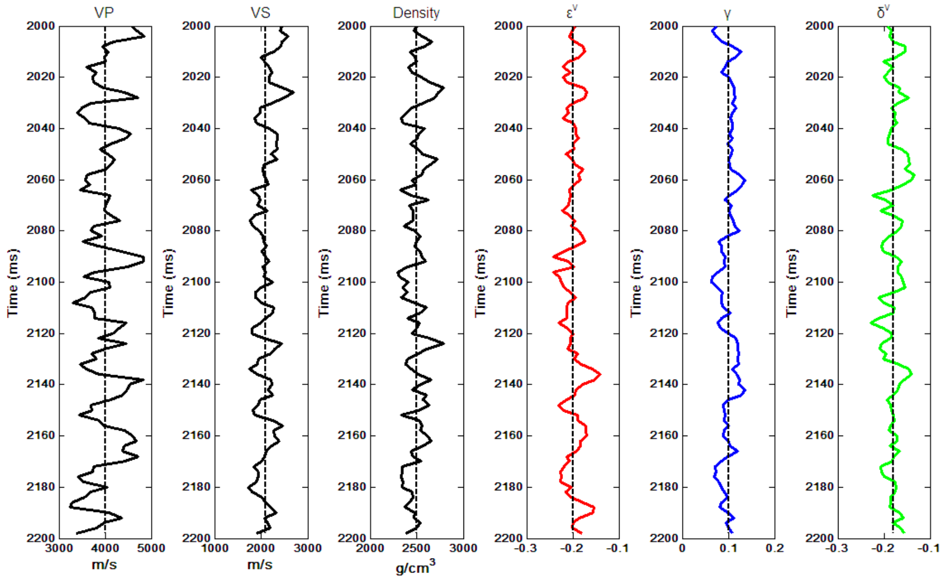

***Figure** 1 The constructed P-wave velocity, S-wave velocity, and density, $\varepsilon^V, \gamma, \delta^V$ in well A (solid lines). The constant background model is dotted.*

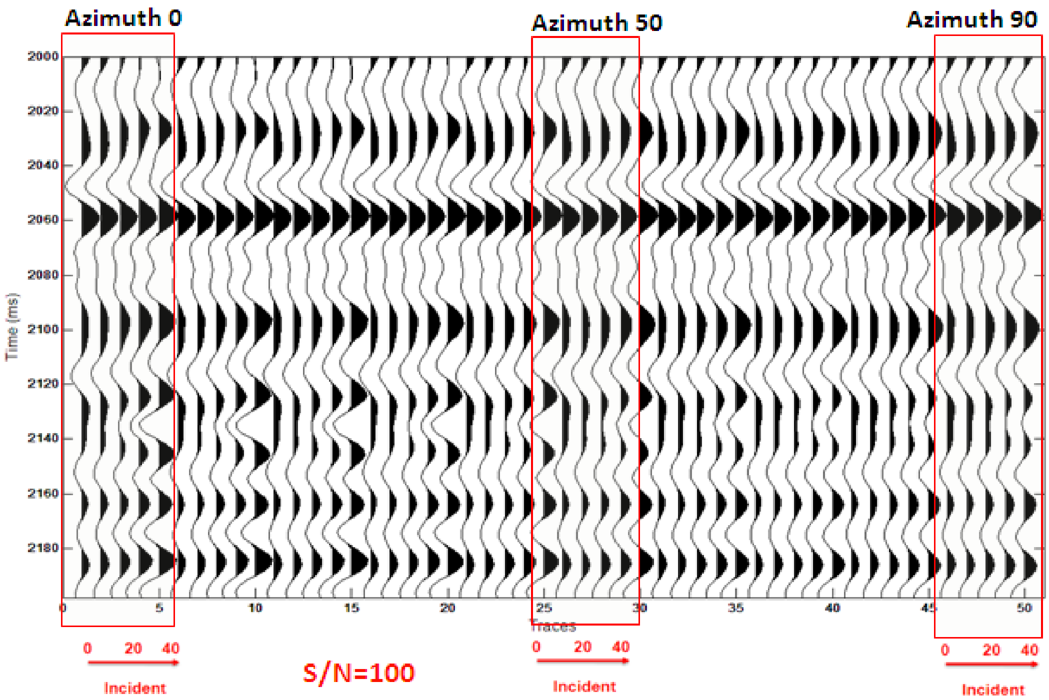

***Figure 2** Synthetic gathers for well model A with different azimuth and Incident angle (S/N=100)*

Conclusions

We proposed a Bayesian AVAZ inversion method by which the posterior distributions for vertical P-wave velocity, vertical S-wave velocity, density, and the anisotropic parameters ε^V, γ and δ^V can be obtained. The synthetic inversion test show that this method can work well with high S/N ratio of seismic data, the three anisotropic parameters are more sensitive to noise level. However, application of this inversion method on real seismic data requires additional information such as geological information, geo-mechanical model and FMI log to constrain prior model and calibrate the inversion results.

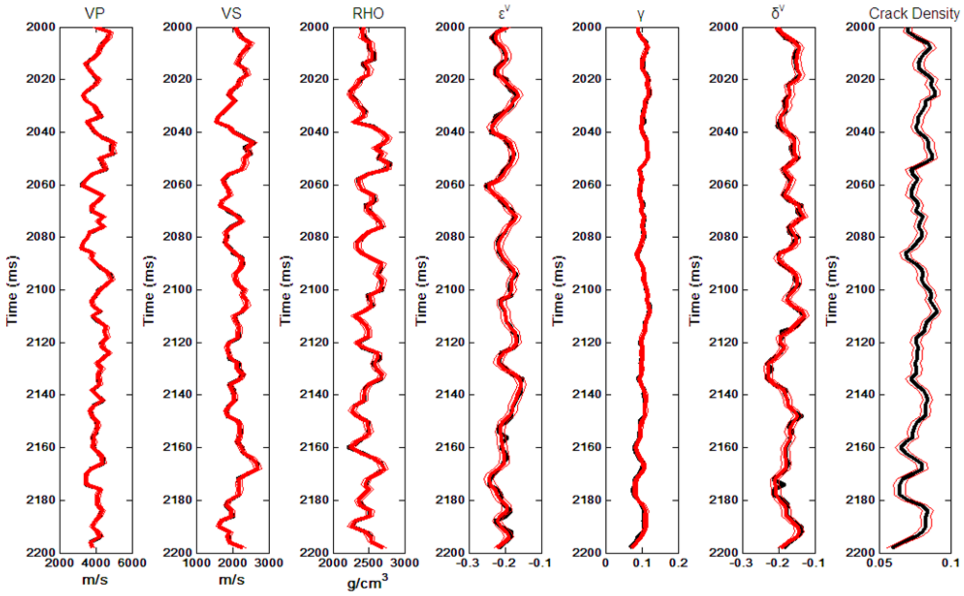

Figure 3 The Maximum a posterior solution (thick red line)of random well model (black line) with S/N ratio 100 and 0.95 prediction interval (thin red lines)

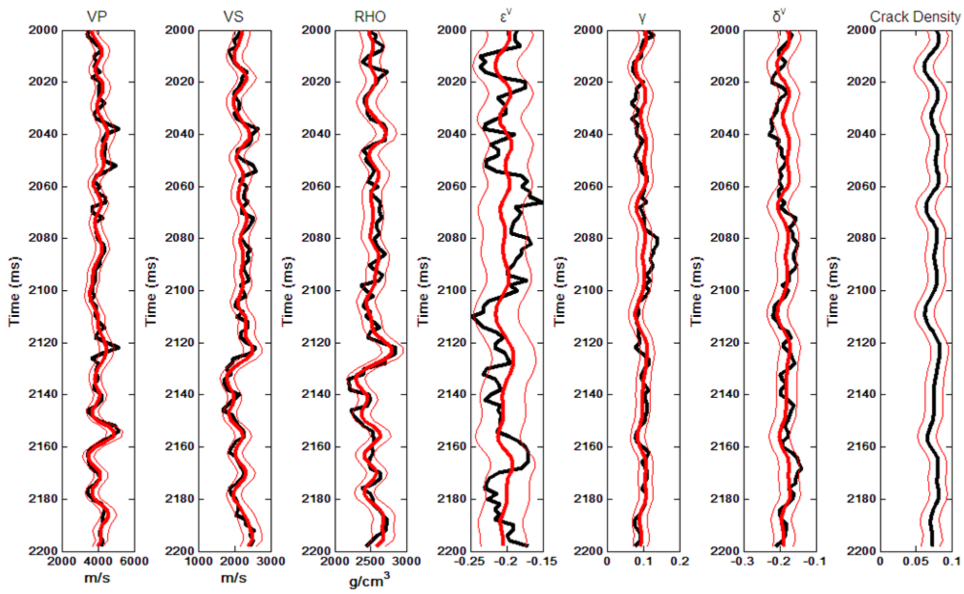

Figure 4 The Maximum a posterior solution (thick red line)of random well model (black line) with S/N ratio 4 and 0.95 prediction interval (thin red lines)

Reference

Bakulin, A., Grechka, V., and Tsvankin, I., 2000, Estimation of fracture parameters from reflection seismic data—Part I: HTI model due to a single fracture set: Geophysics, 65, 1788–1802.

Buland, A., and Omre, H., 2003, Bayesian linearized AVO inversion: Geophysics, 68, 185–198.

Rüger, A., 1997, P-wave reflection coefficients for transversely isotropic models with vertical and horizontal axis of symmetry: Geophysics, 62,713–722.

Tarantola, A., 1987, Inverse problem theory and model parameter estimation: Elsevier Science Publishers.