Seismic characterization of fractured reservoirs: a resolution matrix approach

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Summary

Natural fractures in reservoirs play an important role in determining fluid flow during production, and knowledge of the orientation and density of fractures is required to optimize production. Variations in reflection amplitude with azimuth and incidence angle are sensitive to the presence of fractures. The variation in reflection coefficient of seismic *P*-waves as a function of azimuth and offset for an arbitrary number of differently oriented vertical fractures is analyzed to identify which parameter combinations are well-resolved for various experimental geometries. The results show how to optimize seismic acquisition to help choose the location of infill wells, the orientation of deviated wells, and the relative orientation of neighboring infill wells to ensure adequate drainage.

Introduction

Natural fractures in reservoirs play an important role in determining fluid flow during production, and knowledge of the orientation and density of fractures is required to optimize production from naturally fractured reservoirs (Reiss, 1980; Nelson, 1985). Areas of high fracture density can represent "sweet spots" of high permeability, and it is important to be able to target such locations for infill drilling. Because oriented sets of fractures lead to direction-dependent seismic velocities, the use of seismic waves to determine the orientation of fractures has received much attention. Reflection amplitudes offer advantages over seismic velocities for characterizing fractured reservoirs because they have higher vertical resolution and are more sensitive to the properties of the reservoir.

Current models used to invert the seismic response of fractured reservoirs often assume a single set of perfectly aligned fractures, whereas many reservoirs contain several fracture sets with variable orientation within a given fracture set as illustrated in Figure 1 (see, for example, Gillespie et al., 1993; Sayers, 1998; Sayers and Dean, 2001). In this paper, the variation in the reflection coefficient of seismic *P*-waves as a function of azimuth and offset for an arbitrary number of differently oriented vertical fractures is analyzed to identify which well-resolved parameter combinations are determined for various experimental geometries. The results show how to optimize seismic acquisition to help choose the location of infill wells, the orientation of deviated wells, and the relative orientation of neighboring infill wells to ensure adequate drainage.

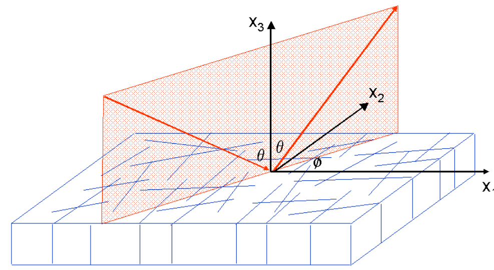


Figure 1. Schematic representation of the reflection of seismic *P*-waves from a fractured reservoir containing vertical fractures with different orientations.

Background theory

Consider the reflection of seismic *P*-waves with angle of incidence θ and azimuth ϕ from a vertically fractured reservoir as shown schematically in Figure 1. The axes x_1, x_2, x_3 are chosen with x_3 perpendicular to the fractured layer. In the neighborhood of the reflection point, the fractured layer is treated as an effective medium with elastic stiffness tensor C_{ijkl} and compliance tensor S_{ijkl}. (Schoenberg and Sayers, 1995). These tensors will vary laterally over the reservoir due, for example, to a lateral variation in fracture density. In the absence of fractures, the elastic stiffness tensor and elastic compliance tensor of the reservoir rock is denoted by C^0_{ijkl} and S^0_{ijkl}, respectively. Sayers and Kachanov (1995) show that the elastic compliance of a fractured reservoir may be written in the form

$$S_{ijkl} = S^0_{ijkl} + \Delta S_{ijkl}, \tag{1}$$

where the excess compliance ΔS_{ijkl} due to the presence of the fractures can be written as

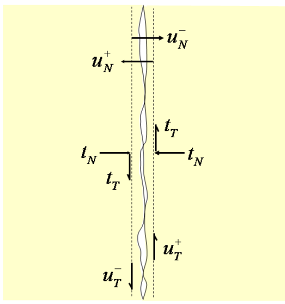
$$\Delta S_{ijkl} = \frac{1}{4}\left(\delta_{ik}\alpha_{jl} + \delta_{il}\alpha_{jk} + \delta_{jk}\alpha_{il} + \delta_{jl}\alpha_{ik}\right) + \beta_{ijkl}. \tag{2}$$

Here, δ_{ij} is the Krönecker delta, α_{ij} is a second-rank tensor, and β_{ijkl} is a fourth-rank tensor defined by

$$\alpha_{ij} = \frac{1}{V}\sum_r B_T^{(r)} n_i^{(r)} n_j^{(r)} A^{(r)}, \tag{3}$$

$$\beta_{ijkl} = \frac{1}{V}\sum_r \left(B_N^{(r)} - B_T^{(r)}\right) n_i^{(r)} n_j^{(r)} n_k^{(r)} n_l^{(r)} A^{(r)}, \tag{4}$$

where the sum is over all fractures in volume V. $n_i^{(r)}$ is the ith component of the normal to the rth fracture, $A^{(r)}$ is the area of the fracture, and $B_N^{(r)}$ and $B_T^{(r)}$ are the normal and shear compliance, respectively, of the rth fracture (Schoenberg, 1980) - see Figure 2.



$$[u_N] = u_N^+ - u_N^-$$
$$[u_T] = u_T^+ - u_T^-$$

$$[u_N] = B_N t_N$$
$$[u_T] = B_T t_T$$

Figure 2. The normal and tangential displacements of the right face of the fracture are denoted by u_N^+ and u_T^+, whereas those of the left face are denoted by u_N^- and u_T^-. The normal and tangential components of the displacement discontinuity at the fracture are given by $[u_N] = u_N^+ - u_N^-$ and $[u_T] = u_T^+ - u_T^-$ and are related to the normal and shear tractions t_N and t_T by the equations shown in the figure.

The shear compliance B_T is assumed to be independent of direction in the plane of the fracture. It should be noted that while the shear compliance of a fracture is expected to be weakly sensitive to any fluid in the fracture, the normal compliance B_N will decrease as the bulk modulus of the fluid increases, in a way that depends on the frequency of the seismic wave, the hydraulic conductance of the fracture, the interconnectivity of the fractures, and the permeability of the background medium. Thus the ratio B_N / B_Y is expected to be sensitive to the fluid in the fractures. Because of interactions between fractures, B_{N} and B_{T} in equations 3 and 4 are functions of the fracture density (Sayers, 2010).

It is assumed in the following that the fractures are vertical and that in the absence of fractures the reservoir is isotropic. For vertical fractures, the elastic symmetry of the fractured rock is then monoclinic, and the non-vanishing components of the excess compliance ΔS_{ijkl} due to the presence of the fractures are (in the conventional two-index notation)

$$\begin{aligned}&\Delta S_{11} = \alpha_{11} + \beta_{1111},\ \Delta S_{22} = \alpha_{22} + \beta_{2222},\ \Delta S_{12} = \Delta S_{21} = \beta_{1122},\\ &\Delta S_{44} = \alpha_{22},\ \Delta S_{55} = \alpha_{11},\ \Delta S_{66} = (\alpha_{11} + \alpha_{22}) + 4\beta_{1122},\\ &\Delta S_{45} = \alpha_{12},\ \Delta S_{16} = \alpha_{12} + 2\beta_{1112}, \text{ and } \Delta S_{26} = \alpha_{12} + 2\beta_{1222}.\end{aligned} \qquad (5)$$

The stiffness tensor of the fractured medium can then be determined by inverting the compliance tensor given by equation 1. This allows the reflection coefficient to be computed for arbitrary fracture density and contrast across the interface using, for example, the method of Schoenberg and Protazio (1992).

In this paper the anisotropy and contrast between the overburden and reservoir is assumed to be small. In this situation, the P-wave reflection coefficient for arbitrary elastic symmetry can be written in the form (Pšenčík and Martins, 2001):

$$\begin{aligned}R_{PP}(\theta,\phi) = {} & R_{PP}^{iso}(\theta) + \frac{1}{2}\Delta\varepsilon_z + \frac{1}{2}\Bigg[\left(\Delta\delta_{\mathrm{x}} - 8\frac{\bar{\upsilon}_S{}^2}{\bar{\upsilon}_P{}^2}\Delta\gamma_{\mathrm{x}}\right)\cos^2\phi + \\ & \left(\Delta\delta_{\mathrm{y}} - 8\frac{\bar{\upsilon}_S{}^2}{\bar{\upsilon}_P{}^2}\Delta\gamma_{\mathrm{y}}\right)\sin^2\phi + 2\left(\Delta\chi_z - 4\frac{\bar{\upsilon}_S{}^2}{\bar{\upsilon}_P{}^2}\Delta\varepsilon_{45}\right)\cos\phi\sin\phi - \\ & \Delta\varepsilon_z\Bigg]\sin^2\theta + \frac{1}{2}\Big[\Delta\varepsilon_{\mathrm{x}}\cos^4\phi + \Delta\varepsilon_{\mathrm{y}}\sin^4\phi + \Delta\delta_z\cos^2\phi\sin^2\phi + \\ & 2\left(\Delta\varepsilon_{16}\cos^2\phi + \Delta\varepsilon_{26}\sin^2\phi\right)\cos\phi\sin\phi\Big]\sin^2\theta\tan^2\theta,\end{aligned} \qquad (6)$$

where $R_{PP}^{iso}(\theta)$ denotes the weak-contrast reflection coefficient at an interface separating two slightly different isotropic media, and the anisotropy parameters ε_{x}, ε_{y}, ε_{z}, ε_{16}, ε_{26}, δ_{x}, δ_{y}, δ_z, χ_z, γ_{x}, γ_{y}, and ε_{45} are given by

$$\begin{aligned}&\varepsilon_{\mathrm{x}} = \frac{A_{11} - \upsilon_P^2}{2\upsilon_P^2},\ \varepsilon_{\mathrm{y}} = \frac{A_{22} - \upsilon_P^2}{2\upsilon_P^2},\ \varepsilon_{\mathrm{z}} = \frac{A_{33} - \upsilon_P^2}{2\upsilon_P^2},\ \varepsilon_{16} = \frac{A_{16}}{\upsilon_P^2},\\ &\varepsilon_{26} = \frac{A_{26}}{\upsilon_P^2},\ \delta_{\mathrm{x}} = \frac{A_{13} + 2A_{55} - \upsilon_P^2}{\upsilon_P^2},\ \delta_{\mathrm{y}} = \frac{A_{23} + 2A_{44} - \upsilon_P^2}{\upsilon_P^2},\\ &\delta_{\mathrm{z}} = \frac{A_{12} + 2A_{66} - \upsilon_P^2}{\upsilon_P^2},\ \chi_{\mathrm{z}} = \frac{A_{36} + 2A_{45}}{\upsilon_P^2}, \gamma_x = \frac{A_{55} - \upsilon_S^2}{2\upsilon_S^2},\\ &\gamma_y = \frac{A_{44} - \upsilon_S^2}{2\upsilon_S^2},\ \varepsilon_{45} = \frac{A_{45}}{\upsilon_S^2}.\end{aligned} \qquad (7)$$

Here $A_{ij} = C_{ij} / \rho$ are the density-normalized elastic stiffnesses, and υ_{P} and υ_{S} are the P- and S-wave velocities in the absence of fractures. The elastic stiffness tensor can be found by inverting the compliance tensor given by equations 1 through 4. The P-wave reflection coefficient $R_{PP}(\theta)$ can be written in the form

$$\begin{aligned}&R_{PP}(\theta,\phi) = R_{PP}^{iso}(\theta) + F_{11}(\theta,\phi)\mu\alpha_{11} + \\ &F_{12}(\theta,\phi)\mu\alpha_{12} + F_{22}(\theta,\phi)\mu\alpha_{22} + F_{1111}(\theta,\phi)\mu\beta_{1111} + \\ &F_{1112}(\theta,\phi)\mu\beta_{1112} + F_{1122}(\theta,\phi)\mu\beta_{1122} + \\ &F_{1222}(\theta,\phi)\mu\beta_{1222} + F_{2222}(\theta,\phi)\mu\beta_{2222}\end{aligned} \qquad (8)$$

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where μ is the shear modulus of the background medium (Sayers, 2009). Analytic expressions for the sensitivities $F_{ij}(\theta,\phi)$ and $F_{ijkl}(\theta,\phi)$ to the α_{ij} and β_{ijkl} are not given here because of space limitations; they are independent of the properties of the fractures. The variation of the sensitivities as a function of incidence angle and azimuth are shown in Figure 3, for fractures added to the Type I gas sand of Kim et al. (1993), with *P*-wave velocity of 4.2 km/s, *S*-wave velocity of 2.7 km/s and density 2.49 g/cc in the absence of fractures. The overburden is assumed to be isotropic with *P*-wave velocity of 3.78 km/s, *S*-wave velocity of 2.43 km/s and density of 2.36 g/cc (see Table 1).

Table 1. Parameters for the example shown in this paper.

	υ_P [km/s]	υ_S [km/s]	ρ [g/cc]
Overburden	3.78	2.43	2.36
Reservoir	4.20	2.70	2.49

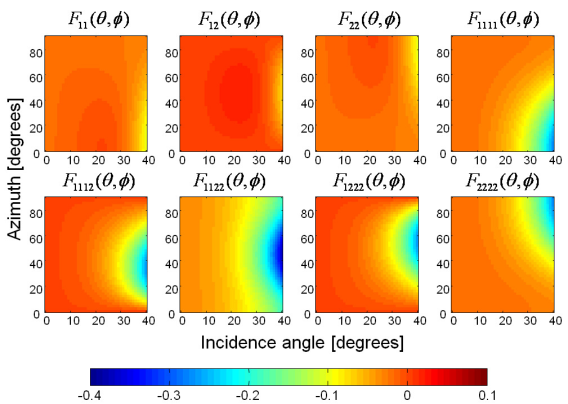

Figure 3. Sensitivities $F_{ij}(\theta,\phi)$ and $F_{ijkl}(\theta,\phi)$ to α_{ij} and β_{ijkl} for the example shown in Table 1.

Acquisition Considerations

In order to determine acquisition requirements for reliable inversion of fracture parameters from AVOA data, different acquisition geometries are considered. Based on model resolution matrices, optimum survey design parameters can be obtained. Typical orthogonal seismic survey designs with different patch geometries (different numbers of live receiver lines) which lead to different azimuth/offset content in acquired seismic data will be compared in the presentation. Figure 4, for example, shows an acquisition geometry that leads to wide azimuth seismic data. For our analysis, we chose receiver and shot line intervals of 200m and receiver and shot station intervals of 50m (bin size of 25 m). In the results reported below, the resolution matrix was analyzed for the case of wide azimuth and long offset surface seismic acquisition by assuming that high quality azimuthally variant data will be sampled for 5 degree steps in azimuth, and 2 degree steps in offset. The resolution matrix was determined using the elastic parameters listed in Table 1.

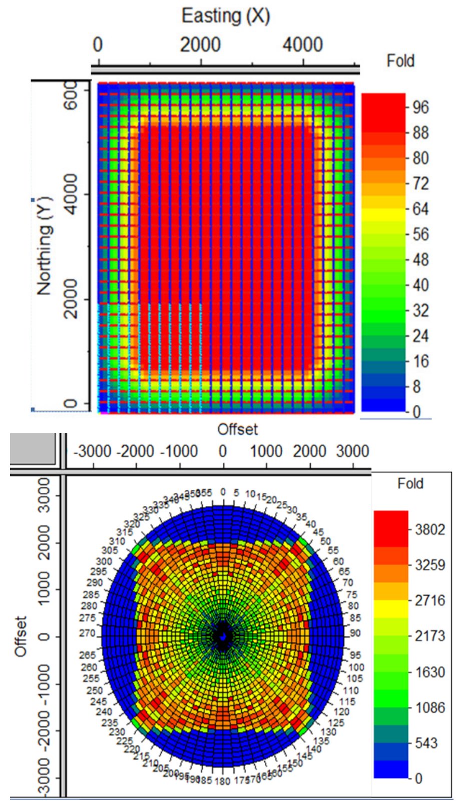

Figure 4. Acquisition geometry leading to wide azimuth data.

Resolution matrix

The forward problem has the simple form $R = F\,w$, where R is a vector of length N containing all measured reflection coefficients, F is an $N \times M$ sensitivity matrix and w is the vector of length M that represents unknown parameters (components α_{ij} and β_{ijkl}). In matrix notation the forward problem has the following form:

$$\begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{bmatrix} = \begin{bmatrix} F_{11_1} & F_{12_1} & F_{22_1} & \cdots & F_{2222_1} \\ F_{11_2} & F_{12_2} & F_{22_2} & \cdots & F_{2222_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ F_{11_N} & F_{12_N} & F_{22_N} & \cdots & F_{2222_N} \end{bmatrix} \begin{bmatrix} \alpha_{11} \\ \alpha_{12} \\ \vdots \\ \beta_{2222} \end{bmatrix} \tag{9}$$

where the vector of unknowns is:

$$\underline{w} = [\alpha_{11}, \alpha_{12}, \alpha_{22}, \beta_{1111}, \beta_{1112}, \beta_{1122}, \beta_{1222}, \beta_{2222}]^T \tag{10}$$

and we assume that the properties of the rock without fractures is known (e.g. from sonic and density logs) and constant. The effect of unknown and variable background properties will be discussed elsewhere. Inversion can be performed using either simple matrix operations, where the solution can be obtained from

$$\underline{w} = \left(\underline{F}^T \underline{F}\right)^{-1} \underline{F}^T \underline{R} \tag{11}$$

or more appropriately using iterative methods such as conjugate gradient. The resolution matrix is a powerful tool for predicting the best resolved parameter combinations for a particular acquisition geometry (see, for example, Menke, 1989). Using singular value decomposition (SVD), the matrix F can be written as the product of three matrices:

$$F = U \Lambda V^T \tag{12}$$

where U is an $N \times N$ matrix of eigenvectors that span the data space and V is an $M \times M$ matrix of eigenvectors that span the model parameters space. Since these eigenvectors are orthonormal, $V^T V = V V^T = I$. Matrix Λ is an $N \times M$ diagonal matrix of non-negative eigenvalues that are called singular values. SVD consists of finding the eigenvalues and eigenvectors of FF^T and $F^T F$. The eigenvectors of $F^T F$ make up the columns of V and the eigenvectors of FF^T make up the columns of U. Also, singular values in diagonal elements of Λ are square roots of eigenvalues from FF^T or $F^T F$ and are arranged in descending order. The singular values are always real numbers. If the matrix F is a real matrix, then U and V are also real.

Some of the singular values may be negligible (close to zero). Therefore Λ is partitioned into a matrix of non-zero singular values and remaining zero elements as

$$\Lambda = \begin{bmatrix} \Lambda_p & 0 \\ 0 & 0 \end{bmatrix} \tag{13}$$

where Λ_p is a $p \times p$ diagonal matrix. The decomposition of F then becomes $F = U \Lambda V^T = U_p \Lambda_p V_p{}^T$ where U_p and V_p consist of the first p columns of U and V respectively. The model resolution matrix is defined as

$$\mathcal{R}_m = V_p V_p{}^T \tag{14}$$

The resolution matrix is formed by zeroing all eigenvalues with a magnitude less than a certain threshold. The diagonal elements of the resolution matrix quantify the resolution of a parameter with larger diagonal element corresponding to a better resolution of the corresponding parameter in inversion. Trade-off between parameters can be determined from the individual rows (or columns) of this matrix. Figure 5 shows the resolution matrix for the wide azimuth acquisition example, where the diagonal elements of the matrix represent the resolution of the α_{ij} and β_{ijkl}. Brighter colors mean better resolution and vice versa.

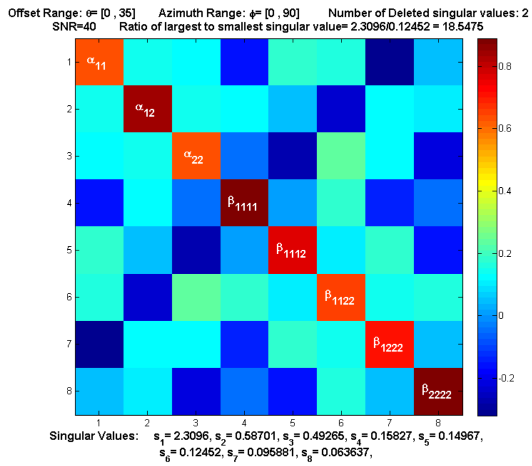

Figure 5. Resolution matrix for the wide azimuth acquisition geometry.

Conclusion

Natural fractures in reservoirs play an important role in determining fluid flow during production, and knowledge of the orientation and density of fractures is required to optimize production. Choosing optimum acquisition parameters is important for the characterization of fractured reservoirs using seismic data. The model resolution matrix is a useful tool for determining acquisition geometry required for reliable inversion of rock parameters from seismic data. It characterizes whether the model parameters can be independently predicted or resolved. In this paper, the significance of different acquisition geometries for inversion of second and fourth rank fracture tensors, using PP amplitude versus offset/azimuth data, are compared and azimuth/offset requirements are presented.

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EDITED REFERENCES

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REFERENCES

Gillespie, P. A., C. B. Howard, J. J. Walsh, and J. Watterson, 1993, Measurement and characterization of spatial distributions of fractures: Tectonophysics, **226**, 113–141, doi:10.1016/0040-1951(93)90114-Y.

Kim, K. Y., K. H. Wrolstad, and F. Aminzadeh, 1993, Effects of transverse isotropy on P-wave AVO for gas sands: Geophysics, **58**, 883–888, doi:10.1190/1.1443472.

Menke, W., 1989, Geophysical data analysis: discrete inverse theory: Academic Press Inc.

Nelson, R. A., 1985, Geologic analysis of naturally fractured reservoirs: Gulf Publishing Company.

Pšenčík, I., and J. L. Martins, 2001, Properties of weak contrast PP reflection/transmission coefficients for weakly anisotropic elastic media: Studia Geophysica et Geodaetica, **45**, 176–199.

Reiss, L. H., 1980, The reservoir engineering aspects of fractured formations: Editions Technip.

Sayers, C. M., 1998, Misalignment of the orientation of fractures and the principal axes for P- and S-waves in rocks containing multiple non-orthogonal fracture sets: Geophysical Journal International, **133**, 459–466, doi:10.1046/j.1365-246X.1998.00507.x.

Sayers, C. M., 2009, Seismic characterization of reservoirs containing multiple fracture sets: Geophysical Prospecting, **57**, 187–192, doi:10.1111/j.1365-2478.2008.00766.x.

Sayers, C. M., 2010, Geophysics under stress: Geomechanical applications of seismic and borehole acoustic waves: SEG Distinguished Instructor Short Course.

Sayers, C. M., and S. Dean, 2001, Azimuth-dependent AVO in reservoirs containing non-orthogonal fracture sets: Geophysical Prospecting, **49**, 100–106, doi:10.1046/j.1365-2478.2001.00236.x.

Sayers, C. M., and M. Kachanov, 1995, Microcrack-induced elastic wave anisotropy of brittle rocks: Journal of Geophysical Research, **100**, 4149–4156, doi:10.1029/94JB03134.

Schoenberg, M., 1980, Elastic wave behavior across linear slip interfaces: The Journal of the Acoustical Society of America, **68**, 1516–1521, doi:10.1121/1.385077.

Schoenberg, M., and J. Protazio, 1992, Zoeppritz rationalized and generalized to anisotropy: Journal of Seismic Exploration, **1**, 125–144.

Schoenberg, M., and C. M. Sayers, 1995, Seismic anisotropy of fractured rock: Geophysics, **60**, 204–211, doi:10.1190/1.1443748.