Some consideration about fluid substitution without shear wave velocity

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Summary

When S-wave velocity is absent, approximate Gassmann equation using P-wave modulus is recommended by Mavko (1995) to calculate the fluid saturation effects. Using both lab data and log data, our study shows that the approximate Gassmann equation might introduce significant error in fluid saturation effect prediction except for unconsolidated rocks. Using S-wave velocity estimated by existing techniques, with same input parameters, the exact Gassmann equation should provide more reliable estimation of fluid saturation effect than that predicted by approximate Gassmann equation.

Introduction

Gassmann equation is regularly used to predict fluid saturation effect in the industry, but it needs input of both compressive wave velocity and shear wave velocity to calculate shear modulus and bulk modulus. Gassmann equation is often applied on log data to model the seismic response due to pore fluid variation. But in practice, quite often the shear wave sonic log is missing for various reasons, especially for old log data. To handle this problem, Mavko (1995) brought up an approximation of Gassmann equation:

$$\frac{M_{sat}}{M_0 - M_{sat}} \approx \frac{M_{dry}}{M_0 - M_{dry}} + \frac{M_f}{\phi\left(M_0 - M_f\right)} \qquad (1)$$

Where M represents the P-wave modulus, and M_{sat}, M_0, M_{dry}, M_f are P-wave modulus of the saturated rock, the mineral, the dry rock and the pore fluid respectively.

Using lab rock physics data measured by Han (1986), we found the absolute velocity error of the predicted P-wave velocity indeed is very small, several percent of the value predicted by exact Gassmann equation. But the velocity change caused by pore fluid change (the fluid saturation effect) might also be only several percent of original velocity before fluid substitution. So this error estimation is misleading, instead we should compare the fluid saturation effects predicted by exact Gassmann equation and approximation Gassmann equation. By doing so we found that the approximate Gassmann equation systematically over estimated the fluid saturation effects, and most often the velocity error introduced by approximate Gassmann is much bigger than the fluid saturation effect predicted by exact Gassmann equation.

Alternatively, we can estimated the shear velocity using existing technique (Greenberg and Castagna, 1992; Xu and White, 1994), and then use exact Gassmann to predict fluid saturation effects. Using both lab data and log data, we found that as long as the the estimated shear wave velocity is not unreasonably unacceptable, it will always provide more reliable estimation of fluid saturation effect than the approximate Gassmann equation.

Fluid substitution with approximate Gassmann equation

In order to check the validity of the approximate Gassmann equation, we first applied it to Han's data. Here we don't consider effects of dispersion, differential pressure, hydration of clay minerals and et al. Thus we choose the data measured at confining pressure of 50 MPa and at 100% water saturation, and then assume 50% of the pore fluid is replaced by gas. The approximate Gassmann equation using only P-wave velocity and exact Gassmann equation using both P-wave and S-wave velocities are applied to calculate the fluid saturation effects respectively. Parameters used in the calculation are listed in Table 1.

Table 1: Parameters used in fluid saturation effect calculation

K_{water}	2.25 GPa	K_{quartz}	37 GPa
ρ_{water}	1.01 g/cc	μ_{quartz}	44 GPa
K_{gas}	0.081 GPa	K_{clay}	21 GPa
ρ_{gas}	0.197 g/cc	μ_{clay}	7 GPa

Figure 1 shows the comparison of P-wave velocity (at Sw=0.5) calculated by the approximate and exact Gassmann equations. The velocity computed by exact Gassmann equation is assumed to be correct and used as reference. The velocities error is defined the difference between the P-wave velocities calculated by exact and approximate Gassmann equation respectively. The mean square error relative to the correct velocity is about 2.23%. From Figure 1, it seems that the approximate Gassmann equation did a good job in estimating the velocity after the saturation has been changed. In Figure 2, we make a blind guess that the P-wave velocity does not change after 50% of the water is replaced by gas. Comparing Figure 1 and 2, we can see that generally our blind guess performs better than the approximate Gassmann equation in estimating the velocity after pore fluid saturation change. Thus Figure 1 is misleading in evaluating the validity of the approximate Gassmann equation.

Fluid substitution without S-wave velocity

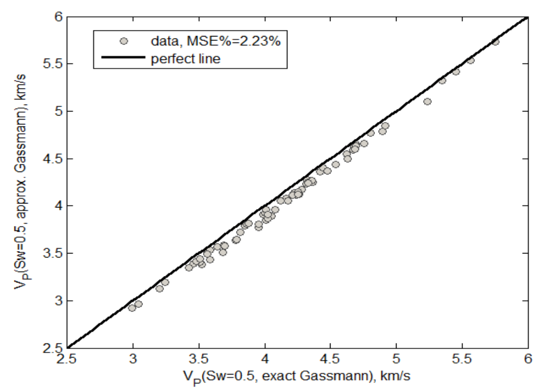

Figure 1: Cross plot between P-wave velocity estimated by exact Gassmann and that estimated by the approximated Gassmann equation (Original water saturation Sw=1.0)

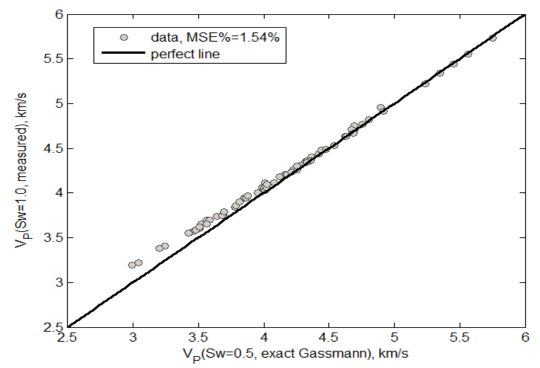

Figure 2: Cross plot between P-wave velocity estimated by exact Gassmann and that by blind guess (assuming no velocity change after saturation is changed to 50%)

Except for unconsolidated rock, the fluid saturation effect of common reservoir rock also lies usually in range of several percent of its original velocity. Thus, to check the validity of the approximate Gassmann equation, we should compare the fluid saturation effects. In Figure 3, we plotted the fluid saturation effect predicted by approximate Gassmann equation against the fluid saturation effect calculated by exact Gassmann. When the saturation is changed from 100% to 50%, for most of the rock samples, the velocity will decrease. The velocity increases slightly for several rock samples because of density effect. All the data points lie below the perfect line, which means that the approximate Gassmann systematically exaggerate the fluid saturation effect. For most of data points which lie below the green line, the approximate Gassmann at least doubles the fluid saturation effect. If the actual fluid saturation effect is -0.05 km/s while the estimated fluid saturation effect is -0.20 km/s, this is a big mistake in fluid saturation effect prediction although the relative velocities error might be small (several percent of the original velocity).

In Figure 4, we plotted the ratio of velocity error (caused by approximation) to fluid saturation effect (calculated by exact Gassmann equation) against porosity, and the color bar shows the original velocity before saturation variation. Among the 70 rock samples, the velocity error introduced by approximation Gassmann equation is bigger than the fluid saturation effect for 74.3% of rock samples. So in term of fluid saturation effect estimation, the approximate Gassmann does not work for most of the rock samples. From Figure 4, it might be said the approximate equation works best for loose sandstones of high porosity and low velocity.

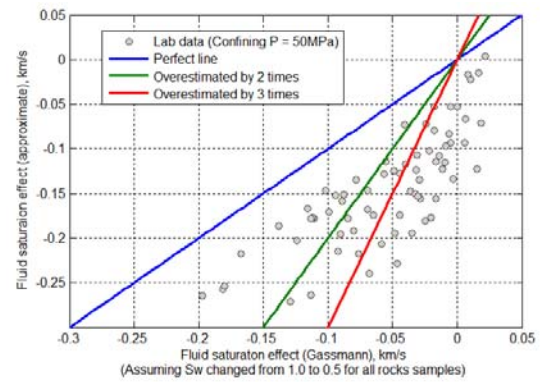

Figure 3: Comparison of fluid saturation effects estimated by exact Gassmann and approximate Gassmann equation

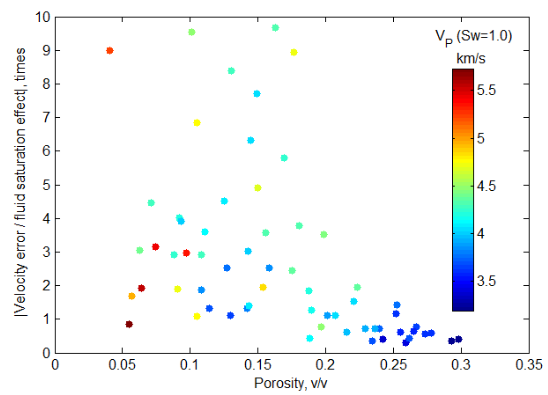

Figure 4: Relation between porosity and relative velocity error by approximate Gassmann equation.

Fluid substitution with estimated S-wave velocity

In stead of using the approximate Gassmann equation, we can also use exact Gassmann and estimated shear wave velocity to predict saturation effect when shear wave velocity is not available. There are two commonly used methods to estimate the shear velocity (Greenberg and Castagna, 1992; Xu and White, 1994), and here we select the technique introduced by Greenberg and Castagana's

(1992) since it requires exactly the same input information as the approximated Gassmann equation (lithology, porosity, saturation, density, elastic moduli and concentrations of constituent minerals and pore fluids). The regression coefficients for pure lithologies used in this study are default and as given by Castagna et al. (1993).

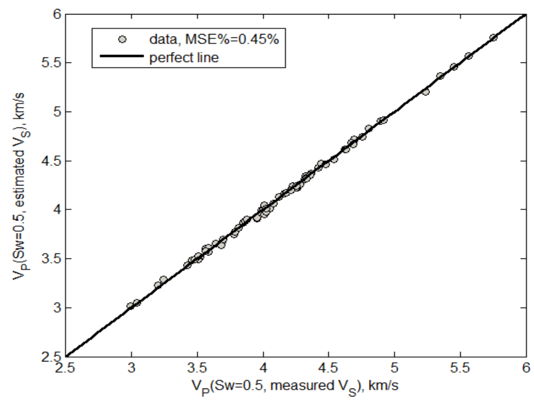

Figure 5: Cross plot between P-wave velocities estimated by exact Gassmann with measured and estimated shear wave velocities (Original water saturation Sw=1.0)

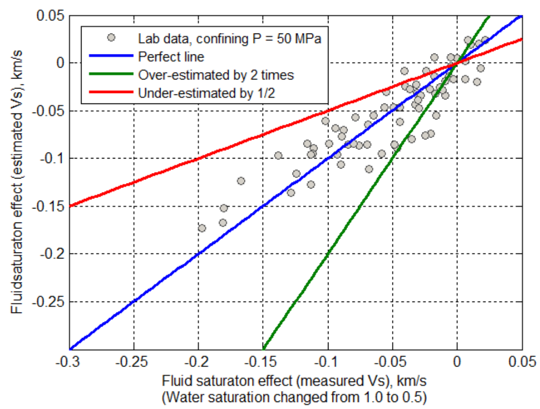

Figure 6: Comparison of fluid saturation effects estimated by exact Gassmann with measured and estimated S-wave velocities respectively

Same as previous study, we assume the water saturation is changed from 100% to 50%, then we use exact Gassmann equation with the estimated shear wave velocity for 100% water saturated rock to compute the P-wave velocities when water saturation is changed to 50%. The result is shown in Figure 5. Comparing with Figure 1, using exact Gassmann and estimated S-wave velocity, the predicted V_P after saturation variation has a much smaller error than that predicted by the approximate Gassmann equation. In term of fluid saturation effect estimation, which we care most, the result (Figure 6) is also much improved as compared to previous study (Figure 3). From Figure 6, the estimated fluid saturation effect rarely double the fluid saturation effect calculated by exact Gassmann equation with measured S-wave velocity except when fluid saturation effect is negligible.

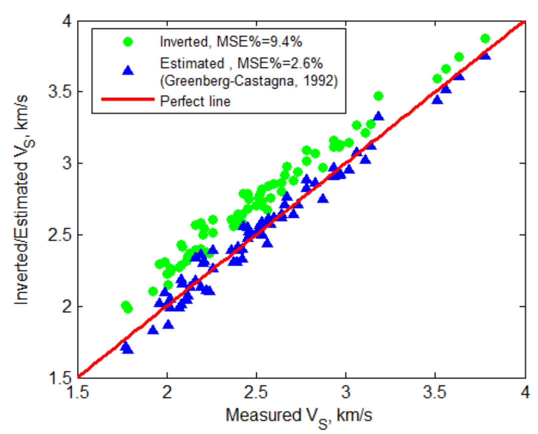

Figure 7: Cross plot between estimated/inverted S-wave velocity and measured S-wave velocity at 100% water saturation

Inversion of S-wave velocity

The approximate Gassmann equation estimates fluid saturation effect without using shear wave velocity information. The same fluid saturation effect can be calculated using exact Gassmann equation, the same P-wave velocity and a certain shear wave velocity. This shear wave velocity is the implicit shear velocity used by the approximate Gassmann equation. The process of finding this shear wave velocity is an inversion process and can be described by the following equation system.

$$\frac{K_{sat1}}{K_0 - K_{sat1}} - \frac{K_{f1}}{\phi(K_0 - K_{f1})} = \frac{K_{sat2}}{K_0 - K_{sat2}} - \frac{K_{f2}}{\phi(K_0 - K_{f2})} \qquad (2)$$

$$K_{sat1} + \frac{4}{3}\mu = \rho_1 V_{P1}^2 \qquad (3)$$

$$K_{sat2} + \frac{4}{3}\mu = \rho_2 V_{P2}^2 \qquad (4)$$

where subscript 1 and 2 represent original water saturation and changed water saturation respectively. K_{sat}, K_0, K_f are bulk modulus of saturated rock, mineral and pore fluid respectively. μ and ρ are shear modulus and bulk density respectively. V_{P1} is the original P-wave velocity, and V_{p2} is the P-wave velocity after the saturation change and is estimated by the approximate Gassmann equation. Basically there are three unknowns (K_{sat1}, K_{sat2}, μ) in three independent equations, so we can solve for μ and calculate the implicit shear wave velocity.

In Figure 7, we plot the measured shear-wave velocity against the shear-wave velocity estimated using method of Greenberg and Castagna (1992) and the shear-wave velocity inverted from fluid saturation effect predicted by approximate Gassmann equation. This plot explains why the approximate Gassmann equation doest not work very well in fluid saturation effect prediction: it implicitly and systematically use shear wave velocity much higher than the actual shear-wave velocity. For this data set, as long as the average relative error of the estimated shear-wave velocity is not higher than 9.4%, it will provide more reliable result of fluid saturation effect estimation.

Fluid substation of log data

Figure 8 shows example of fluid substitution of a tight gas reservoir. The porosity of the gas reservoir lies around 20%. The gas saturation is not high and is mostly lower than 25%. We assume that the gas are all replaced by brine and want to see the saturation effect on P-wave velocity. The fluid properties are calculated using FLAG program with in-situ pressure and temperature. The blue curves are supplied log data. The estimated shear wave velocity (green curve) is calculated by method of Greenberg and Castagna (1992) with regression coefficients for pure lithologies given by Castagna et al. (1993).

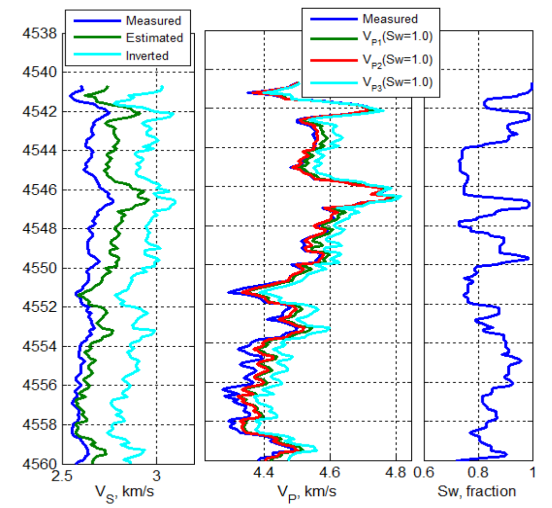

Figure 8: Comparison of fluid saturation effect predictions: (1) V_{P1}, calculated by exact Gassmann with estimated V_S; (2) V_{P2}, calculated by exact Gassmann with measured V_S ; and (3) V_{P3}, calculated by approximate Gassmann

With estimated S-wave velocity, supplied P-wave log data, using exact Gassmann equation we can calculate P-wave velocity (V_{P1}) when water saturation is changed to 100% for the whole gas reservoir interval. With supplied S-wave log and P-wave log data, using exact Gassmann equation, the calculated P-wave velocity at 100% water saturation is V_{P2}. Using P-wave log data only and the approximate Gassmann equation, the calculated P-wave velocity at 100% water saturation is V_{P3}. Comparing V_{P2} with the original P-wave log data, we can see that fluid saturation effect is mostly negligible to small. The saturation effect estimated by estimated shear wave velocity is very close to that predicted by using exact Gassmann equation and supplied P-wave and S-wave log data. The approximate Gassmann equation always magnifies the saturation effect. For example, at depth interval of 4542-4546 m, the fluid saturation effect is almost negligible, but the approximate Gassmann equation predicts a noticeable fluid saturation effect. The overall saturation effect of this gas reservoir interval might be negligible, but with approximate Gassmann, you might predict a significant fluid saturation effect and thus provide false information for seismic forward modeling.

As introduced in previous section, we can invert the S-wave velocity implicitly used by approximate Gassmann equation. As shown in the Figure 8, the inverted S-wave lies far away from the measured S-wave log data, the average error is more than 10% percent, as long as the estimated S-wave velocity lies within the error bar defined by the inverted S-wave, it will give more reliable fluid saturation effect prediction than the approximate Gassmann equation.

Conclusion

Based on analysis of both lab data and log data, our study shows that the approximate Gassmann equation (equation 1) is not a proper approximation of Gassmann to predict fluid saturation effect except for unconsolidated rocks. Using S-wave velocity estimated by existing techniques, with same input parameters, the exact Gassmann equation should provide more reliable estimation of fluid saturation effect than that predicted by approximate Gassmann equation.

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EDITED REFERENCES

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